MATH253X-UX1
Spring 2019

Midterm Exam 2

Name: ____________________

Instructions. (100 points) You have 120 minutes. Closed book, closed notes, no calculator. *Show all your work* in order to receive full credit.

(6^{pts}) **1.** Show that $\displaystyle\lim_{(x,y)\to(-2,1)} \frac{x+y+1}{xy+2}$ does not exist.

(10^{pts}) **2.** Let $w = \dfrac{xy}{x-z}$.

(a) (4 pts) Verify that w satisfies the partial differential equation $xw_x + xw_z = yw_y$.

(b) (6 pts) Use the appropriate chain rule to find w_s for $(s,t) = (2,1)$ if $x = s^2t$, $y = t^2 - s$, $z = 3t$.

(16pts) **3.** Consider the surface $z = \frac{2}{3}x^{\frac{3}{2}} + 2y$ over the rectangular region $R = [1, 4] \times [0, 1]$.

(a) (8 pts) Compute the volume under the surface and over R.

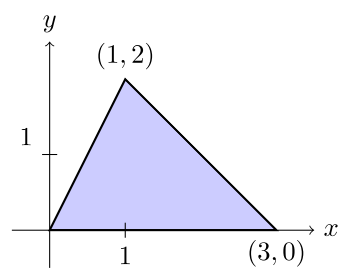
(b) (8 pts) Compute the surface area of $z = \frac{2}{3}x^{\frac{3}{2}} + 2y$ over the region R.

(8pts) **4.** Find an equation of the tangent plane at $(2, 0, 1)$ to the surface

$$x^2 z - yz^2 + y^2 = 4.$$

(6pts) **5.** Let $z = \ln(xy)$. Use the total differential to approximate Δz when moving from the point $(1, 2)$ to the point $(0.98, 2.1)$.

(16pts) **6.** Assume a planar lamina has density $\rho = x$ and occupies the following region:



(a) (8 pts) Give two equivalent expressions for the mass of the lamina first setting up bounds and integrand in $dx\,dy$ then in $dy\,dx$. DO NOT evaluate.

(b) (8 pts) Compute M_x the moment of mass with respect to the x-axis for the lamina.

(12^{pts}) **7.** Find and classify all critical points of

$$f(x, y) = x^3 + xy^2 - 4xy + x + 1.$$

(10^{pts}) **8.** Find the absolute minimum and maximum of

$$f(x, y) = x^2 - y^2 + 3x$$

in the region $x^2 + 2y^2 \leq 4$.

(8pts) **9.** Fully SET UP bounds and integrand in polar coordinates to represent the volume of the solid bounded by the cone $z = 2 - \sqrt{x^2 + y^2}$ and the inverted paraboloid $z = 8 - x^2 - y^2$. DO NOT evaluate.

(8pts) **10.** Let

$$f(x, y) = x^2 y + \sin(\pi y).$$

(a) (5 pts) Find the directional derivative of f at $(1, -1/2)$ in the direction of $\langle -3, 4 \rangle$.

(b) (3 pts) What is the maximum rate of change of f at the point $(1, -1/2)$?