

Math 651 – Supplementary homework exercise

Q1: (The Characteristic Property of the Quotient Topology is characteristic.)
Let $p : X \rightarrow Y$ be a surjective function, where X is a topological space.
Consider two topologies on Y , denoting the resulting topological spaces
by Y_1, Y_2 , and the functions $p_i : X \rightarrow Y_i$

Suppose both Y_1 and Y_2 have the property that for every topological space
 Z and every $f : Y_i \rightarrow Z$, that f is continuous if, and only if, $f \circ p_i : X \rightarrow Z$
is. Show that $Y_1 = Y_2$.

$$\begin{array}{ccc} X & & \\ \downarrow p_i & \searrow f \circ p_i & \\ Y_i & \xrightarrow{f} & Z \end{array}$$

Thus, since the quotient topology has this property, it is the unique topol-
ogy on Y with this property.