Mathematics 405: Abstract Algebra

Spring 2018

Instructor: John Rhodes

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Office Hours: M 9:15-10:15; W 12:00-1:00; R 9:00-10:00;

Web page: `https://jarhodesuaf.github.io/M405.html`

Prerequisites: Math 265; Engl 111; Engl 211 or 213

Credit Hours: 3.0

Textbook: Contemporary Abstract Algebra, by Joseph Gallian, 9th edition, Cengage

Class Meetings: TR 11:30-1:00, in Chapman 106

Exams: Midterm: March 6, in class;
Final: 10:15–12:15, Thursday, May 3

Course Overview and Goals:

"Algebra" is that part of mathematics in which we combine objects (e.g., numbers, vectors, matrices, polynomials,. . .) by operations (e.g., addition, multiplication,. . .) which obey certain rules (e.g., commutativity, associativity,. . .). Abstract Algebra looks beyond the specifics of the objects and operations, at their more fundamental properties and the implications of those properties. The resulting knowledge is surprisingly broad, spanning not only the algebraic topics you are familiar with from earlier coursework, but also providing tools for applications ranging from geometric symmetry to physics to cryptography.

A first course in Abstract Algebra introduces the student to three fundamental algebraic structures: *groups*, *rings*, and *fields*. A *group* is a set of objects together with a single binary operation $\cdot$ so that any two group elements can be 'multiplied' to create a new group element (closure under multiplication); multiplication is associative; there exists a distinguished element 1 (the identity element) so that for all $g \in G$, $g \cdot 1 = 1 \cdot g = g$; and each element has an inverse (for all $g \in G$, there exits g^{-1} so that $g^{-1} \cdot g = g \cdot g^{-1} = 1$). As familiar examples, you might keep in mind the integers together with addition, $(\mathbb{Z}, +)$, the positive real numbers with multiplication, $(\mathbb{R}^+, \times)$, or invertible 2×2 real matrices with matrix multiplication. The term 'group' was first used by Galois around 1830, the modern conception was introduced by Weber and von Dyck in the late 1880s, and these ideas gained wide-spread acceptance only in the twentieth century.

After an introduction to groups, we study *rings* and *fields*, which are sets with *two* binary operations. While you will find you already have seen examples of these, by taking an abstract approach we can gain much deeper understanding.

This course makes heavy use of a formal approach to mathematics: We state definitions and provide proofs of implications of those definitions. While we consider many examples to build intuition and see how abstract ideas relate to the concrete and familiar, formulating and proving theorems is the central focus. Thus skills you picked up in Math 265 will be essential, and you will grow much more comfortable with their use.

Course Mechanics:

Class meetings will be run as interactive lectures. While I will be presenting material at the board, and you will be taking notes, I will also be asking for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers to these, but I do expect you to be actively following and participating — that makes the class more interesting for us all. I welcome questions as we go along.

Class attendance is expected, although I will not formally take roll. If you miss a class, you should get notes from another student. Homework assignments will be posted on the course web page, and you should make it a habit to check for new problems after each class.

Homework will be assigned daily, and due at class the following Tuesday. At the beginning of class, there may be a little time for simple questions on homework, but you should expect to get your homework questions answered during office hours. Because of the emphasis on proof-writing, grading will take into account not only correctness but also clarity of exposition. Expect to spend time and effort in produce lucid proofs.

I encourage you to work with others on the homework, and to share ideas for solutions, but you must *write up solutions independently*. You will learn nothing from simply copying a solution. Even though you may find you can't do every problem, you must make a reasonable attempt on them all. The entire homework assignment will be checked to be sure you have attempted everything. Selected problems will be graded completely.

Homework will be accepted without penalty until 4 pm on its due date, either at my office or in my mailbox in the math department office. Beyond this, I will not accept *any* late homework that has not been cleared ahead of time or is not due to a genuine emergency (e.g., a death in the family).

Examinations: The midterm exam will be 1.5 hours in length. You will need to state and understand definitions and theorems, be familiar with examples, and prove some relatively straight-forward statements.

The final examination will consist of two parts: an in-class part that will focus on definitions, examples, and 'routine' proofs, and a take-home part that will consist of more challenging proofs which you will be able to work on for at least several days. For the take-home part you will be able to refer to your textbook, class notes, and homework, but nothing else.

Any form of cheating on these exams will be dealt with harshly. At a minimum, the full examination (take-home and in-class) will receive a score of zero. Depending on my concern with the extent of cheating, any incident may result in a course grade of F, and I may also request a University hearing which could result in suspension or expulsion. Please note that evidence of collaboration on work in mathematics is usually obvious, so even if your personal honor is worth nothing to you, cheating is a foolish risk to take.

Missed examinations that are not approved in advance will result in a zero on that exam. No make-up exams will be given except in extreme circumstances (e.g., family death, documented illness, etc.). Notifying me by email or a note that you will miss an exam is not sufficient for advance approval; you must speak with me to be excused.

Auditing of this course will only be allowed for those who agree to attend regularly, submit homework, and take the midterm exam.

Course Grades will be assigned using the following weights:

Homework	30%
Midterm	30%
Final Exam	40%

Letter grade bands are: 90 – 100% A; 80 – 89% B; 70 – 79% C; 60 – 69 D; 0 – 59% F. Scores near the ends of the bands will receive a $\pm$. I may adjust bands (but only downward) if I feel that is necessary for fairness. A stellar grade on the final may also overcome earlier weaker work and improve a student's final grade beyond the straight weighting scheme.

Other Policies:

Course accommodations: If you need course adaptations or accommodations because of a disability, please inform your instructor during the first week of the semester, after consulting with the Office of Disability Services.

University and Department Policies: Your work in this course is governed by the UAF Student Code of Conduct. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams at `http://www.dms.uaf.edu/dms/Policies.html`.

Tentative Schedule:

Week	Chapter
Jan 15 — 19	0, 1
Jan 22 — 26	2, 3
Jan 29 — Feb 2	4, 5
Feb 5 — 9	6,7
Feb 12 — 16	7,8
Feb 19 — 23	9,10
Feb 26 — Mar 2	10,11
Mar 5 — 9	Midterm Exam, 12
Mar 12 — 16	Spring Break
Mar 19 – 23	13, 14
Mar 26 — 30	15
Apr 2 — 6	16, 17
Apr 9 — 13	19
Apr 16 — 20	20, 21
Apr 23 — 27	22
May 3	Final Exam