Quiz 7 | Name: Solutions

Math 314 F01 | 12/3/14

1. Suppose B is a real symmetric 4×4 matrix with 4 distinct eigenvalues. For each of the following, indicate whether S *could possibly* be a matrix of eigenvectors of B, and briefly give a reason why.

(a) $S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{pmatrix}$

cannot be matrix of eigenvectors, since the columns are not orthogonal

(b) $S = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

could be matrix of eigenvectors since the columns are orthogonal (though not normalized)

2. Consider the quadratic form $f(x, y, z) = x^2 + 4y^2 + 6z^2 - 2xy + 4xz$.

(a) With $\mathbf{x} = (x, y, z)$, give a symmetric matrix A so that

$$f(x, y, z) = \mathbf{x}^T A \mathbf{x}.$$

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 4 & 0 \\ 2 & 0 & 6 \end{pmatrix}$$

(b) Is $\mathbf{x}^T A \mathbf{x} > 0$ for every $\mathbf{x} \neq \mathbf{0}$? Show enough work to justify your answer. The 3 determinants we check are:

$|1| = 1 > 0$

$\begin{vmatrix} 1 & -1 \\ -1 & 4 \end{vmatrix} = 4 - 1 = 3 > 0$

$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 4 & 0 \\ 2 & 0 & 6 \end{vmatrix} = 2 \begin{vmatrix} -1 & 4 \\ 2 & 0 \end{vmatrix} + 6 \begin{vmatrix} 1 & -1 \\ -1 & 4 \end{vmatrix} = 2(-8) + 6(3) = -16 + 18 = 2 > 0$

Thus A is positive definite, and $\vec{x}^T A \vec{x} > 0$ for all $\vec{x} \neq \vec{0}$