Quiz 6
Math 314 F01

Name: Solutions
11/07/14

1. If the Gram-Schmidt process is performed to orthonormalize the 3 vectors

$$\mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix},\ \mathbf{b}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix},\ \mathbf{b}_3 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}.$$

there is no need to modify the direction of $\mathbf{b}_2$ since it is already orthogonal to $\mathbf{b}_1$. What vector $\mathbf{q}_3$ must replace $\mathbf{b}_3$? (Be sure you normalize.)

$\vec{w}_1 = \vec{b}_1$
$\vec{w}_2 = \vec{b}_2$

$$\vec{w}_3 = \vec{b}_3 - \mathrm{proj}_{\vec{w}_1}\vec{b}_3 - \mathrm{proj}_{\vec{w}_2}\vec{b}_3 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} - \frac{3}{3}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -1/3 \\ -2/3 \\ 0 \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\vec{q}_3 = \frac{1}{\|(1,1,2,0)\|}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \\ 0 \end{pmatrix}$$

2. Compute the following determinant *using elimination.* Show all your work.

$$\begin{vmatrix} 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 1 & 1 & 3 & 4 \end{vmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & -1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 1 & 1 & 3 & 4 \end{pmatrix} \xrightarrow[\text{exchg}]{\text{row}} \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 0 & -1 \\ 0 & 2 & 0 & 2 \\ 1 & 1 & 3 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 0 & 0 & -1 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 1 \end{pmatrix} \xrightarrow[\text{exchg}]{\text{row}} \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 1 & 3 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{so } |A| = (-1)^2 \cdot 1 \cdot 2 \cdot (-2) \cdot (-1) = 4$$