Name: Solutions

10/31/14

1. Let $\mathbf{a} = (1, -1, 0)$

(a) Find a matrix P that can be used to project vectors in $\mathbb{R}^3$ onto the line spanned by $\mathbf{a}$. Simplify your answer completely.

$$P = \frac{\vec{a}\vec{a}^T}{\vec{a}^T\vec{a}} = \frac{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}(1 \;\; -1 \;\; 0)}{(1 \;\; -1 \;\; 0)\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}} = \frac{1}{1+1}\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) Find the projection of $(1, -2, 3)$ onto the line spanned by $\mathbf{a}$.

$$P\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + 1 \\ -\frac{1}{2} - 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ -\frac{3}{2} \\ 0 \end{pmatrix}$$

2. The matrix $P = \begin{pmatrix} 5/6 & 1/3 & -1/6 \\ 1/3 & 1/3 & 1/3 \\ -1/6 & 1/3 & 5/6 \end{pmatrix}$ projects onto a subspace V of $\mathbb{R}^3$. Give a matrix that projects onto $V^\perp$.

$$I - P = \begin{pmatrix} \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \end{pmatrix}$$