

Consider the vectors

$$\mathbf{a} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

1. Two of these vectors are orthogonal to each other. Which ones? Indicate how you can tell.

$$\vec{b} \cdot \vec{c} = 2(-1) + 0 \cdot 2 + 1 \cdot 2 = -2 + 2 = 0$$

$$\text{so } \vec{b} \perp \vec{c}$$

2. The third vector above can be written as a linear combination of the two that are orthogonal. Do so.

$$\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = u \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + v \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

From the 2nd coordinate $2 = u \cdot 0 + v \cdot 2$ so $v = 1$

Then from the 1st coord $-3 = u \cdot 2 + 1(-1)$ so $u = -1$

$$\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = (-1) \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

3. What do the statements in questions (1) and (2) above tell you about the set of all vectors of the form $eu + dv + ew$. Is this a line, plane, or 3-space?

$\vec{r} = s\vec{b} + t\vec{c}$
It is a plane.