Math 314 Name :__________________

Exam 2 November 21, 2014

Show your work on all problems.

1. (15 pts.) A subspace V of $\mathbb{R}^4$ has basis $\mathbf{v}_1 = (1, 0, -1, 1), \mathbf{v}_2 = (0, 1, -2, 1), \mathbf{v}_3 = (-1, 0, 0, 1)$. Find an orthonormal basis for V.

2. (12 pts.–4 pts. each) Four data points from an experiment are

x	-1	0	1	2
y	2	1	3	7

(a) Give a matrix equation that you would *like* to solve (but that probably doesn't have a solution) to find a quadratic of the form $y = ax^2 + bx + c$ that passes through the 4 data points.

(b) Give a matrix equation that *could be* solved to find the least-squares best-fit quadratic for (a). (Do not simplify or solve.)

(c) Briefly explain the key idea behind passing from your equation in (a) to that in (b). (Any good answer will use the word "projection").

3. (12 pts.) In $\mathbb{R}^4$, a subspace W is spanned by $(1, 0, 1, 0)$ and $(0, 2, 1, 1)$. Find a basis for $W^{\perp}$.

4. (14 pts.) Let $A = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -1 & 0 \\ -2 & 1 & -1 \end{pmatrix}$.

   (a) (10 pts.) Compute $|A|$ *by elimination*. (No other methods will receive credit).

   (b) (4 pts.) Give the upper right (row 1, column 3) entry of A^{-1}.

5. (12 pts.) A matrix $A = \begin{pmatrix} .8 & .2 \\ .3 & .7 \end{pmatrix}$ has eigenvectors $(1, 1)$ and $(2, -3)$, with respective eigenvalues 1 and 0.5.

   (a) (5 pts.) Give a diagonalization $A = S\Lambda S^{-1}$ of the matrix.

   (b) (7 pts.) Use your diagonalization to compute $\lim_{k\to\infty} A^k$.

6. (14 pts.–7 pts. each) Let $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find an eigenvector for the *smallest* eigenvalue of A.

7. (6 pts.) Give a formula for the value of y in the solution to:

$$\begin{aligned} 2x - y \quad +z &= 1 \\ -x + 2y \qquad &= 2 \\ 3x + y \ +2z &= 1 \end{aligned}$$

   (Do not simplify your answer.)

8. (15 pts.–3 pts.each) Complete the following:

   (a) If P is a permutation matrix, then det P is ______________. (*Give all possibilities.*)

   (b) If V is a k-dimensional subspace of $\mathbb{R}^n$, then $V^\perp$ is _______-dimensional.

   (c) If P is a projection matrix then $P^2 =$ _______.

   (d) If a "warped box" (parallelepiped) in 3-d has edges given by the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, then its volume is most easily computed as...

   (e) If A is 4×4 with $|A| = -3$, then $|-A| =$ ______________, $|A^T| =$ ______________, and $|A^{-1}| =$ ______________.

   (f) The "big formula" for the determinant of an $n \times n$ matrix is a sum and difference of ______________ terms, each of which is a product of ______________ entries of the matrix.

   (g) If the columns of A are independent, the formula for a matrix that projects onto the column space of A is ....