Math 314 Name :_____

Exam 2 April 4, 2008

1. (15 pts. – 5 pts. each) A matrix A and its reduced row echelon form U are:

$$A = \begin{pmatrix} 1 & 0 & -1 & 1 & 1 \\ 3 & 1 & -1 & 4 & 2 \\ 1 & 1 & 1 & 2 & 0 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 2 & -3 \\ 4 & -3 & -10 & 1 & 7 \end{pmatrix}, \qquad U = \begin{pmatrix} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

   (a) Give a basis for the nullspace of A.

   (b) Give a basis for the rowspace of A.

   (c) Give a basis for the columnspace of A.

2. (6 pts.)What is the rank of the matrix $B = \begin{pmatrix} 1 & -3 & 2 & 3 \\ 2 & 3 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 3 & -6 & 7 & 8 \end{pmatrix}$? Show your work.

3. (12 pts.) Let $A = \begin{pmatrix} 0 & -2 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{pmatrix}$.

   (a) (6 pts.) Compute $|A|$ using elimination. (No credit for other methods.)

   (b) (2 pts.) What is $|A^{-1}|$?

   (c) (2 pts.) What is $|2A|$?

   (d) (2 pts.) What is $|A^4|$?

4. (9 pts.) Use Cramer's rule to solve the system of equations $\begin{cases} 2x + 3y = 1, \\ 7x + 11y = 3. \end{cases}$ (No credit for other methods.)

5. (10 pts.) Find the inverse of $C = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & -2 & 1 \end{pmatrix}$ using the formula involving cofactors. (No credit for other methods.)

6. (6 pts.) For a computer graphics system, the point (x, y) in the plane $\mathbb{R}^2$ is encoded by $(x, y, 1)$ so that 3×3 matrices can be used to rotate and translate points. Give a 3×3 matrix that will produce the combined action of first translating by 2 units in the y-direction, and then rotating by $90°$ clockwise.

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7. (12 pts. – 4 pts. each) For the vector space $\mathbb{P}_5$ of polynomials of degree at most 5, answer the following.

   (a) Is the subset of $\mathbb{P}_5$ whose elements are those polynomials with a quadratic term of $2x^2$ a subspace? Explain why or why not. NOT RELEVANT FOR SPRING 2013

   (b) Is the subset of $\mathbb{P}_5$ whose elements are those polynomials with no quadratic term a subspace? Explain why or why not. NOT RELEVANT FOR SPRING 2013

   (c) What is the dimension of $\mathbb{P}_5$? Why? NOT RELEVANT FOR SPRING 2013

8. (12 pts. – 6 pts. each) Consider the bases $\mathcal{B} = \{(1, -2), (3, 1)\}$ and $\mathcal{C} = \{(-2, 1), (3, 7)\}$ of $\mathbb{R}^2$.

   (a) If $\mathbf{v} = (-1, -5)$, find the coordinates $[\mathbf{v}]_{\mathcal{B}}$ of $\mathbf{v}$ with respect to $\mathcal{B}$. NOT RELEVANT FOR SPRING 2013

   (b) Give a matrix $P_{\mathcal{B}\to\mathcal{C}}$ such that $[\mathbf{x}]_{\mathcal{C}} = P_{\mathcal{B}\to\mathcal{C}}[\mathbf{x}]_{\mathcal{B}}$ for all vectors $\mathbf{x} \in \mathbb{R}^2$. NOT RELEVANT FOR SPRING 2013

9. (18 pts. – 6 pts. each) Give short answers, with explanations.

(a) An 18×23 matrix A has a 7-dimensional nullspace. Will $A\mathbf{x} = \mathbf{b}$ be solvable for every $\mathbf{b} \in \mathbb{R}^{18}$? Briefly explain your reasoning.

(b) If A is an $m \times n$ matrix and the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ is onto, what can you say about the rank of A? Briefly explain your reasoning. NOT RELEVANT FOR SPRING 2013

(c) Suppose A is a non-zero 3×3 matrix, with rows $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. If $\det A = 0$, then what are the possible dimensions of $\text{Span}\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$? (List all possibilities.) Briefly explain your reasoning.