Math 314 Name :__________________

Exam 1 October 20, 2014

Show your work on all problems.

1. (19 pts.) A matrix A has LU factorization given by

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Answer the following questions about A. (You should *not* need to multiple L and U to get A).

(a) (3 pts.) Describe exactly the 3rd elimination step that is performed on A to reduce it to echelon form.

(b) (4 pts.) Are there vectors $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has no solution? Explain your reasoning.

(c) (7 pts.) Give a basis for the nullspace of A.

(d) (5 pts.) For $\mathbf{b} = (1, 3, 1)$, a solution to $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = (1, 1, 0, 0)$. Could this be the only solution? If not, give all solutions.

2. (12 pts. – 6 pts. each) If A is $m \times n$, then

   (a) $A\mathbf{x} = \mathbf{b}$ will be solvable for every $\mathbf{b}$ if the (circle one) *column space* / *nullspace* of A is ________. This happens when the rank of A is ________.

   (b) $A\mathbf{x} = \mathbf{b}$ will have at most one solution if the (circle one) *column space* / *nullspace* of A is ________. This happens when the rank of A is ________.

3. (13 pts.) For

$$A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & -1 & 0 \\ 2 & 0 & 1 \end{pmatrix},$$

   either find A^{-1}, or show it does not exist.

4. (10 pts.) Determine whether the following vectors in $\mathbb{R}^4$ are independent.

$$(1, 0, -1, 1), (-1, 1, 2, 1), (1, 1, 2, 3)$$

5. (18 pts. – 4 pts. each M, 2 pts. each M^{-1}) Give matrices M that perform the following operation to a 3×3 matrix A when MA is computed. Also, give M^{-1}.

   (a) Reorder the rows of A so that the 3rd is on top, the 1st in the middle, and the 2nd at the bottom:

   $M =$ $\qquad\qquad$ $M^{-1} =$

   (b) Multiply rows 1, 2, and 3 by the scalars 2, 4, and 8, respectively:

   $M =$ $\qquad\qquad$ $M^{-1} =$

   (c) Add twice the first row to the third:

   $M =$ $\qquad\qquad$ $M^{-1} =$

6. (10 pts. – 2 pts. each) Suppose A is 4×3, and when $\mathbf{b} = (-1, 1, 2, -2)$ the solutions to $A\mathbf{x} = \mathbf{b}$ form a line. Then

   (a) The solutions (circle one) *form / do not form* a subspace of $\mathbb{R}^3$.

   (b) The rank of A must be ________.

   (c) For a randomly chosen $\mathbf{b}$ in $\mathbb{R}^4$ the problem $A\mathbf{x} = \mathbf{b}$ (circle one) *will have / probably will have / probably will not have / will not have* a solution.

   (d) The nullspace of A is a (circle one) *point / line / plane / 3-space/ 4-space.*

   (e) The row space of A is a (circle one) *point / line / plane / 3-space/ 4-space.*

7. (10 pts.) Find the LDL^T factorization of $A = \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$.

8. (8 pts. – 4 pts. each) Short answers:

   (a) Give the formula for the inverse of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

   (b) If the LU factorization of an $n \times n$ matrix A is known, then $A\mathbf{x} = \mathbf{b}$ can be solved by solving what triangular systems?