Math 314 Name :__________________

Exam 1 February 27, 2013

1. (12 pts.) Find the inverse of the following matrix (or show none exists). Show all your work.

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 2 \\ -1 & 0 & 2 \end{pmatrix}$$

2. (26 pts.) Consider the matrix equation $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & -2 & 2 \\ 2 & 2 & -1 & -2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

(a) (13 pts.) Find all solutions, showing your work.

(b) (3 pts.) Do the solutions to this problem form a subspace of $\mathbb{R}^4$? (CIRCLE ONE) Yes / No

(c) (6 pts.) Based on your answer to part (a), give a basis for the nullspace of A.

(d) (4 pts.) If $\mathbf{b}$ were changed to be a different vector in $\mathbb{R}^3$, then $A\mathbf{x} = \mathbf{b}$ will (CIRCLE ONE)
(1) certainly be solvable,
(2) probably be solvable, but may not be,
(3) probably not be solvable, but may be,
(4) certainly not be solvable.

3. (12 pts.) Suppose a 3×5 matrix A has an LU factorization with

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$$

(a) (9 pts.) Describe, in order, each of the elimination steps that were performed on A to reach the echelon form U.

(b) (3 pts.) Can you say what the rank of A must be? If so, what is it? If not, explain why not enough information has been given.

4. (21 pts. – 3 pts. each) Give short answers.

(a) For a matrix to have an inverse, it must be $n \times n$ with rank ________.

(b) The formula for the inverse of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is:

(c) The simple formula for the inverse of a permutation matrix P is $P^{-1} =$ __________

(d) If an $m \times n$ matrix A with $m \geq n$ is randomly chosen, its rank is virtually certain to be ________.

(e) If A is 7×13 with rank 5, then its nullspace will have dimension ________,

(f) ... and its columnspace will have dimension ________.

(g) If A is $n \times n$, and there is some $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has no solutions, then for any $\mathbf{c}$, the number of solutions to $A\mathbf{x} = \mathbf{c}$ must be one of the following: (CIRCLE ALL THAT APPLY)
none / one / infinitely many

5. (12 pts. – 4 pts. each) Give matrices M that perform the following actions on a 3×4 matrix A when the product MA is computed.

   (a) reorder the rows of A, so that the top one goes to the bottom, and the others move up by one each.

   (b) add -3 times the 2nd row to the third

   (c) multiply the 3rd row by -7

6. (17 pts.) Suppose you are given 5 vectors, $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ in $\mathbb{R}^5$, and create a 5×5 matrix A that has the $\mathbf{v}_i$ as its columns, in order. The MATLAB command `R=rref(A)` then produces the output:

$$R = \begin{pmatrix} 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

   (a) (8 pts.) Are the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ independent? Explain, by referring to the *definition* of independence.

   (b) (3 pts.) What is the dimension of the span of the vectors?

   (c) (6 pts.) Give a basis for the span of the vectors.