Class solutions
March 26, 2020

Math 265, Homework due 2/23/2020

§8, #14 Version 1

Theorem 1. *If A, B and C are sets, then $(A \vee B) - C = (A - C) \vee (B - C)$.*

Proof. In order to prove that these sets are equal, we will show that they are subsets of one another.

First, to prove that $(A \vee B) - C \in (A - C) \vee (B - C)$, we will consider an element $x \in (A \vee B) - C$. Then, we know that $x \in A \vee x \in B$, and that $x \notin C$. This means that, whether x is in A or B, it is definitely not in C, or $(A - C) \vee (B - C)$. Thus, we have proven that $(A \vee B) - C \in (A - C) \vee (B - C)$.

Next, to prove that $(A - C) \vee (B - C) \in (A \vee B) - C$, we will let $y \in (A - C) \vee (B - C)$. Then, we know that y is an element of A and not C, or y is an element of B and not C. Since, in both of these cases, y is not an element of C, we know that y is never an element of C, although y can be an element of A or B. Equivalently, in logical terms, our set can be described as $\{y | (y \in A \cup B) \wedge (y \notin C)\}$, or $(A \vee B) - C$.

This proves that $(A \vee B) - C = (A - C) \vee (B - C)$. □

§8, #14 Version 2

Theorem 2. *If A, B and C are sets, then $(A \cup B) - C = (A - C) \cup (B - C)$.*

Proof. Suppose $x \in (A \cup B) - C$. Thus, $x \in A$ or $x \in B$ by definition of union. Also, $x \notin C$ by definition of set subtraction.

Therefore $x \in A - C$ or $x \in B - C$.

Consequently, $(A - C) \cup (B - C)$.

This implies that $(A \cup B) - C = (A - C) \cup (B - C)$.

□

§8, #14 Version 3

Theorem 3. *If A, B, and C are sets, then $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.*

Proof. First, to see $(A \cup B) \setminus C \subseteq (A \setminus C) \cup (B \setminus C)$ let $a \in (A \cup B) \setminus C$. Thus, $a \in A \cup B$ and $a \notin C$. So $a \in A$ or $a \in B$ and $a \notin C$. Then $a \in A$ and $a \notin C$, or $a \in B$ and $a \notin C$. Hence, $a \in A \setminus C$ or $a \in b \setminus C$. Therefore, $a \in (A \setminus C) \cup (B \setminus C)$.

Second, to see $(A \setminus C) \cup (B \setminus C) \subseteq (A \cup B) \setminus C$ let $a \in (A \setminus C) \cup (B \setminus C)$. Then $a \in A \setminus C$ or $a \in B \setminus C$. Furthermore, $a \in A$ and $a \notin C$, or $a \in B$ and $a \notin C$. Thus, $a \in A$ or $a \in B$ and $a \notin C$. Subsequently, $a \in A \cup B$ and $a \notin C$. So $a \in (A \cup B) \setminus C$.

Therefore, $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$. □

§8, #14 Version 4

Theorem 4. *If A, B, C are sets, then $(A \cup B) - C = (A - C) \cup (B - C)$.*

Proof. Observe $(A \cup B) - C = \{x : (x \in A \cup B) \wedge (x \notin C)\}$

$$= \{x : (x \in A) \vee (x \in B) \wedge (x \notin C)\}$$

$$= \{x : (x \in A) \vee (x \in B) \wedge (x \notin C) \wedge (x \notin C)\}$$

$$= \{x : (x \in A \wedge x \notin C)\} \cup \{x : (x \in B \wedge x \notin C)\}$$

$$= (A - C) \cup (B - C)$$

□