Quiz 8
Math 253

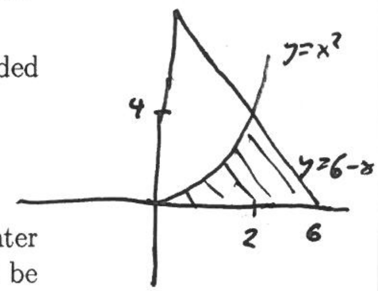
Name: Key
10/23/19

1. A metal plate is shaped like the region in the first quadrant bounded by

$$y = 0, \quad y = 6 - x, \quad \text{and} \quad y = x^2,$$

with mass density $\rho(x, y) = y^2 \;\; kg/m^2$.

Give an expression involving integrals for the x-coordinate of its center of mass. You do not need to evaluate the integrals, but they should be fully set up, with appropriate limits of integration.



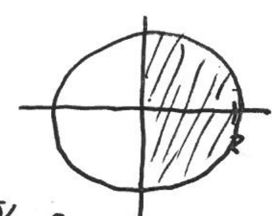
$$\bar{x} = \frac{\iint_D x y^2 \, dA}{\iint_D y^2 \, dA} = \frac{\int_0^4 \int_{\sqrt{y}}^{6-y} x y^2 \, dx\, dy}{\int_0^4 \int_{\sqrt{y}}^{6-y} y^2 \, dx\, dy}$$

$y = x^2 \rightarrow x = \sqrt{y}$

$y = 6 - x \rightarrow x = 6 - y$

2. Draw the region of integration for the following integral, and use that to convert to polar coordinates and evaluate fully:

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx$$



$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 (r^2)^{3/2} \, r \, dr \, d\theta = \int_{-\pi/2}^{\pi/2} \int_0^2 r^4 \, dr \, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \left. \frac{r^5}{5} \right|_{r=0}^{2} d\theta = \int_{-\pi/2}^{\pi/2} \frac{32}{5} \, d\theta = \pi \frac{32}{5}$$