Quiz 6
Math 253

Name: Key

10/09/19

1. Use the chain rule to compute $\frac{\partial g}{\partial u}(1,3)$ if

$$g(u,v) = f(uv, u^2+v^2)$$

assuming g, f and the partial derivatives of f have the values given in the table:

	g	f	f_x	f_y
$(1,3)$	2	-2	2	3
$(3,10)$	-1	3	-2	-1

$g(u,v) = f(x(u,v), y(u,v))$ where $x(u,v) = uv$, $y(u,v) = u^2+v^2$

$\frac{\partial x}{\partial u} = v$

$\frac{\partial y}{\partial u} = 2u$

$$\left.\frac{\partial g}{\partial u}\right|_{(1,3)} = \left.\frac{\partial f}{\partial x}\right|_{x(1,3),y(1,3)} \cdot \left.\frac{\partial x}{\partial u}\right|_{(1,3)} + \left.\frac{\partial f}{\partial y}\right|_{x(1,3),y(1,3)} \cdot \left.\frac{\partial y}{\partial u}\right|_{(1,3)}$$

$$= \left.\frac{\partial f}{\partial x}\right|_{(3,10)} \left. v\right|_{(1,3)} + \left.\frac{\partial f}{\partial y}\right|_{(3,10)} \left. 2u\right|_{(1,3)} = (-2)\cdot 3 + (-1)\cdot 2 = -8$$

2. Compute the directional derivative of the function

$$g(x,y,z) = \frac{x}{y} + xz^2 + \ln(z+x)$$

at the point $(-2,1,3)$ in the direction toward the origin.

$\nabla g = < \frac{1}{y} + z^2 + \frac{1}{z+x}, -\frac{x}{y^2}, 2xz + \frac{1}{z+x} >$

$\nabla g(-2,1,3) = < 11, 2, -11 >$

$\vec{u} = \frac{-<-2,1,3>}{\sqrt{4+1+9}} = \frac{1}{\sqrt{14}} < 2,-1,-3 >$

$D_{\vec{u}} g = < 11, 2, -11 > \cdot \frac{1}{\sqrt{14}} < 2,-1,-3 > = \frac{53}{\sqrt{14}}$