

Quiz 5
Math 253

Name: Key
10/01/19

1. Compute $\frac{\partial f}{\partial x}(-1, 2)$ and $\frac{\partial f}{\partial y}(-1, 2)$ for the function

$$f(x, y) = e^{-x^2} \cos(\pi y) + \frac{3}{1 + xy^2}$$

$$\begin{aligned} \frac{\partial f}{\partial x}(-1, 2) &= -2xe^{-x^2} \cos(\pi y) - \frac{3y^2}{(1+xy^2)^2} \Big|_{(-1, 2)} \\ &= 2e^{-1} \cos(2\pi) - \frac{3 \cdot 4}{(1-4)^2} \\ &= \frac{2}{e} - \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y}(-1, 2) &= e^{-x^2} (-\sin(\pi y) \pi) - \frac{3(2xy)}{(1+xy^2)^2} \Big|_{(-1, 2)} \\ &= e^{-1} (-\pi \sin(2\pi)) + \frac{3 \cdot 4}{(1-4)^2} = \frac{4}{3} \end{aligned}$$

2. Explain why the following limit does *not* exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2 + y^2}$$

Taking the limit along the x -axis (so $y=0$)

$$\frac{y^2}{x^2 + y^2} = \frac{0}{x^2} = 0 \rightarrow 0$$

Taking the limit along the y -axis (so $x=0$)

$$\frac{y^2}{x^2 + y^2} = \frac{y^2}{y^2} = 1 \rightarrow 1$$

Since these limits are different, there is no one value that $\frac{y^2}{x^2 + y^2}$ approaches.