Quiz 4
Math 253

Name: Key
09/17/19

1. Consider the parametrized curve in the plane

$$\mathbf{r}(t) = \langle t + \cos(\pi t),\ \ln(1+t) \rangle.$$

Find the unit vector $\mathbf{T}$ tangent to the curve at the point $(0, \ln 2)$.

$r(0) = \langle 0, \ln 2 \rangle$ when $\ln(1+t) = \ln 2$, i.e. $t = 1$

$r'(t) = \langle 1 - \pi \sin(\pi t),\ \frac{1}{1+t} \rangle$

$r'(1) = \langle 1 - \pi \sin(\pi),\ \frac{1}{2} \rangle = \langle 1, \frac{1}{2} \rangle$

Tangent direction is $\langle 1, \frac{1}{2} \rangle$ or $\langle 2, 1 \rangle$

So $T = \frac{1}{\sqrt{5}} \langle 2, 1 \rangle = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

2. A rocket in space moves with acceleration

$$\mathbf{a}(t) = \langle t,\ 1,\ t^{-1} \rangle\ m/sec^2.$$

At time $t = 2$ it has velocity $\langle 2, 1, 0 \rangle\ m/sec$.

Determine its velocity as a function of time. Indicate appropriate units.

$\vec{v}(2) = \langle 2, 1, 0 \rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \left\langle \frac{t^2}{2}, t, \ln t \right\rangle + \langle a, b, c \rangle$

$\langle 2, 1, 0 \rangle = \vec{v}(2) = \langle 2, 2, \ln 2 \rangle + \langle a, b, c \rangle$

$a = 0,\ b = -1,\ c = -\ln 2$

$\vec{v}(t) = \left\langle \frac{t^2}{2}, t-1, \ln t - \ln 2 \right\rangle \frac{m}{sec}$