 Solutions

Table of Indefinite Integrals

$\int cf(x)\,dx = c\int f(x)\,dx$	$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
$\int k\,dx = kx + C$	
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int e^x\,dx = e^x + C$	$\int b^x\,dx = \frac{b^x}{\ln b} + C$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

1. For the following integrals, decide if you would use a u-substitution. If so, *just write down the u-substitution*. If not, *evaluate the integral.*

(a) $\displaystyle\int e^{\cos x}\sin x\,dx =$ $u = \cos x$

(b) $\displaystyle\int \frac{dx}{ax+b} =$ $u = ax+b$, or guess $\frac{1}{a}\ln\lvert ax+b\rvert + C$

(c) $\displaystyle\int_0^2 \lvert 2x-1\rvert\,dx =$ $u = 2x-1$ (or none if done graphically)

(d) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} =$ $u = \ln x$

(e) $\displaystyle\int (7x - 7^{-x})\,dx =$ $u = -x$

(f) $\displaystyle\int_0^1 x(\sqrt[3]{x}+\sqrt[4]{x})\,dx =$ none, multiply out (see other side)

(g) $\displaystyle\int \pi\,dt =$ none $\pi t + C$

(h) $\displaystyle\int \frac{3\,dr}{\sqrt{1-r^2}} =$ none, $3\arcsin(r) + C$

(i) $\displaystyle\int \tan^2\theta\sec^2\theta\,d\theta =$ $u = \tan\theta$

(j) $\displaystyle\int \frac{dx}{(1+x^2)\tan^{-1}(x)} =$ $u = \arctan\theta$

2. Complete the u-substitution, or any other work, for the integrals from problem 1.

(a)

$$\int e^{\cos x}\sin x\,dx = -\int e^u\,du = -e^u + C = -e^{\cos x} + C$$

$u = \cos x \quad du = -\sin x\,dx$

(b)

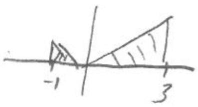
$$\int \frac{dx}{ax+b} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + C = \frac{1}{a}\ln|ax+b| + c$$

$u = ax+b \quad du = a\,dx$

(c)

$$\int_0^2 |2x-1|\,dx = \frac{1}{2}\int_{-1}^{3} |u|\,du = \frac{1}{2}\left(\frac{1}{2}+\frac{9}{2}\right) = \frac{5}{2}$$

$u = 2x-1 \quad du = 2\,dx$



(d)

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{1}{\sqrt{u}}\,du = 2u^{1/2}\Big|_1^4 = 2(2-1) = 2$$

$u = \ln x \quad du = \frac{1}{x}dx$

(e)

$$\int 7x - 7^{-x}\,dx = \frac{7}{2}x^2 - \int 7^{-x}\,dx = \frac{7}{2}x^2 + \int 7^u\,du = \frac{7}{2}x^2 + (\ln 7)7^u + C$$

$u = -x$

$du = -dx$

$$= \frac{7}{2}x^2 + (\ln 7)7^{-x} + C$$

(f)

$$\int_0^1 x\left(\sqrt[3]{x} + \sqrt[4]{x}\right)dx = \int_0^1 \left(x^{4/3} + x^{5/4}\right)dx = \frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4}\Big|_0^1 = \frac{3}{7} + \frac{4}{9} = \frac{27+28}{63} = \frac{55}{63}$$

(g)

$$\int \pi\,dt = \pi t + C$$

(h)

$$\int \frac{3\,dr}{\sqrt{1-r^2}} = 3\arcsin(r) + C$$

(i)

$$\int \tan^2\theta\sec^2\theta\,d\theta = \int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3\theta}{3} + C$$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta$

(j)

$$\int \frac{dx}{(1+x^2)\tan^{-1}x} = \int \frac{du}{u} = \ln|u| + C = \ln|\tan^{-1}(x)| + C$$

$u = \tan^{-1}(x) \quad du = \frac{1}{1+x^2}$