Math F251: Section 5.5 Worksheet Solutions

**1.** Evaluate the integral by making the given substitution.

**(a)** $u = \sin\theta$: $du = \cos\theta\, d\theta$

$$\int \sin^2\theta \cos\theta \, d\theta = \int u^2 du = \frac{u^3}{3} + C = \frac{\sin^3\theta}{3} + C$$

**(b)** $u = x^4 - 5$: $du = 4x^3 dx$ $\quad x^3 dx = \frac{1}{4} du$

$$\int \frac{x^3}{x^4 - 5} \, dx = \int \frac{1}{u}\left(\frac{1}{4}\right) du = \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|x^4 - 5| + C$$

**2.** Evaluate the indefinite integral by substitution. What should you choose as $u$?:

$$\int e^x \sqrt{1 + e^x} \, dx = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C$$

$u = 1 + e^x$

$du = e^x dx$

$$= \frac{2}{3}(1 + e^x)^{3/2} + C$$

**3.** Evaluate the indefinite integrals:

**(a)**

$$\int 5^t \sin(5^t)\, dt = \int \frac{1}{\ln(5)} \sin u \, du = \frac{-1}{\ln(5)} \cos u + C$$

$u = 5^t$

$du = (\ln 5) 5^t \, dt$

$\frac{1}{\ln(5)} du = 5^t \, dt$

$$= -\frac{1}{\ln 5} \cos(5^t) + C$$

**(b)**

$$\int \frac{x}{1+x^4}\, dx = \int \frac{1}{2} \frac{1}{1+u^2} du = \frac{1}{2} \arctan(u) + C$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$= \frac{1}{2} \arctan(x^2) + C$$

**4.** Evaluate the definite integrals:

**(a)**

note change of limits!

$$\int_0^1 (3t-1)^{50}\, dt = \int_{-1}^{2} \frac{1}{3} u^{50}\, du = \frac{1}{3 \cdot 51} u^{51} \Big|_{-1}^{2} = \frac{1}{153}(2^{51}+1)$$

$u = 3t-1$

$du = 3\,dt$

$\frac{1}{3} du = dt$

**(b)**

$$\int_0^{\pi/2} \cos x \sin(\sin(x))\, dx = \int_0^1 \sin u \, du = -\cos u \Big|_0^1 = -\cos(1) + \cos 0$$

$u = \sin x$

$du = \cos x \, dx$

$$= 1 - \cos(1)$$