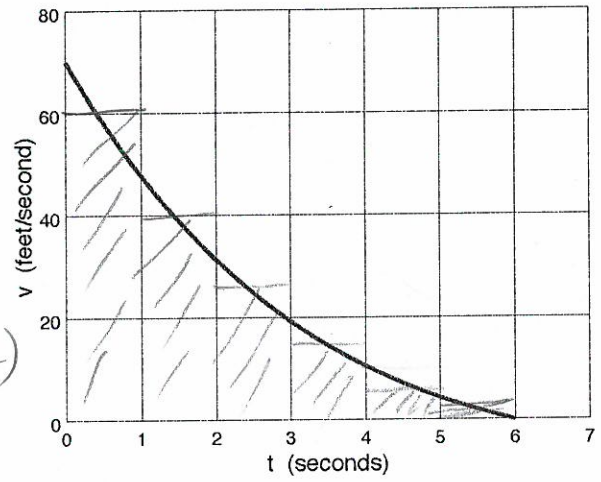
1. The velocity graph $v(t)$ of a braking car is shown.

(a) Use the graph to estimate the distance traveled by the car when the brakes are applied. (*Suggestion: Use 3 or 6 rectangles.*)



$\approx 60 \cdot 1 + 40 \cdot 1 + 27 \cdot 1 + 17 \cdot 1 + 7 \cdot 1 + 2 \cdot 1 \left(\frac{ft}{sec}\right) \cdot (sec)$

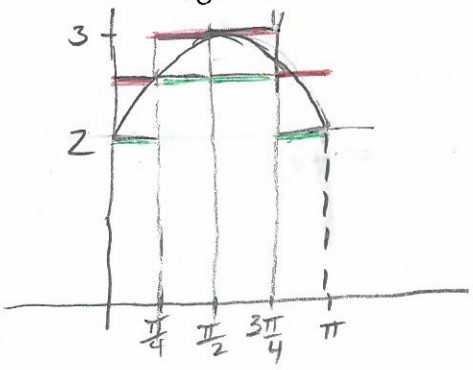
$= 153$ ft

(Answers will vary

(b) Write the exact distance as a definite integral.

$$\int_0^6 v(t)\,dt$$

2. Evaluate the upper and lower sums for $f(x) = 2 + \sin x$ on $0 \le x \le \pi$ with $n = 4$. Illustrate with a diagram.



$\text{Upper}_4 = f\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{4} + f\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{4} + f\left(\frac{3\pi}{4}\right)\cdot\frac{\pi}{4}$ (red)

$= \left(2+\frac{1}{\sqrt{2}}\right)\frac{\pi}{4} + 3\cdot\frac{\pi}{4} + 3\cdot\frac{\pi}{4} + \left(2+\frac{1}{\sqrt{2}}\right)\frac{\pi}{4}$

$= (10+\sqrt{2})\frac{\pi}{4}$

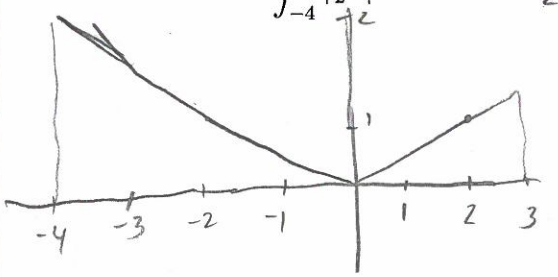
$\text{Lower}_4 = f(0)\cdot\frac{\pi}{4} + f\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{4} + f\left(\frac{3\pi}{4}\right)\cdot\frac{\pi}{4} + f(\pi)\cdot\pi/4$ (green)

$= 2\cdot\frac{\pi}{4} + \left(2+\frac{1}{\sqrt{2}}\right)\cdot\frac{\pi}{4} + \left(2+\frac{1}{\sqrt{2}}\right)\frac{\pi}{4} + 2\cdot\frac{\pi}{4}$

$= (8+\sqrt{2})\frac{\pi}{4}$

3. Evaluate the integral by interpreting it in terms of areas. (*Hint: Start by sketching the integrand.*)

$$\int_{-4}^{3} \left|\tfrac{1}{2}x\right| dx \quad = \tfrac{1}{2}(4)(2) + \tfrac{1}{2}(3)\left(\tfrac{3}{2}\right) = 4 + \tfrac{9}{4} = \tfrac{25}{4}$$

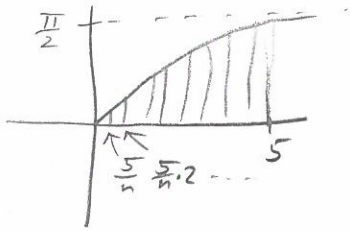


4. (a) Set up an expression for the following integral as a limit of sums; you will not be able to compute the limit:

$$\int_0^5 \arctan x \, dx \quad = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\Delta x = \frac{5-0}{n} = \frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{5i}{n}\right)\frac{5}{n}$$



(b) Using a graph of $y = \arctan x$, sketch a diagram which shows that

$$\frac{5\arctan 5}{2} \leq \int_0^5 \arctan x \, dx \leq \frac{5\pi}{2}$$

↑ area of triangle

↑ area of rectangle