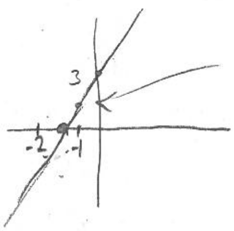
Math F251: Section 4.7 Worksheet Solutions

1. Find the point on the line $y = 2x + 3$ which is closest to the origin.



(x, y), distance to origin is $\sqrt{x^2+y^2} = \sqrt{x^2+(2x+3)^2}$

$= \sqrt{5x^2+12x+9}$ $\quad x \in (-\infty, \infty)$

The minimum of this occurs at the same pt the square of the distance has a minimum, so let

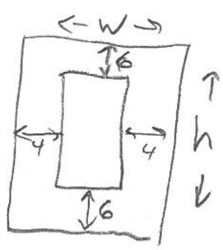
$f(x) = 5x^2+12x+9$

$f'(x) = 10x+12 = 0$

Critical pt must be a minimum

$x = \frac{-6}{5} = -1.2$, $\quad y = 2\left(\frac{-6}{5}\right)+3 = \frac{3}{5} = .6$

2. The top and bottom margins of a poster are each 6 cm and the side margins are 4 cm. If the area of the printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest total area.



$(w-8)(h-12) = 384 \qquad h = 12 + \frac{384}{w-8}$

Minimize $A = hw \qquad w > 8$

$$A(w) = \left(12 + \frac{384}{w-8}\right)w = 12w + \frac{384w}{w-8}$$

$$A'(w) = 12 + \frac{384(w-8) - 384w(1)}{(w-8)^2}$$

$$= 12 - \frac{8(384)}{(w-8)^2}$$

$A'(w)$ DNE at $w=8$ (but $w>8$ so that does not matter)

$A'(w) = 0$ if

$$12 = \frac{8(384)}{(w-8)^2}$$

$$(w-8)^2 = \frac{8(384)}{12} = 256$$

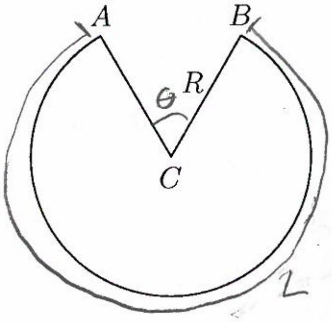
$$w - 8 = 16$$

$$w = 24$$

Since $A''(w) = \frac{8(384)2}{(w-8)^3} > 0$, this is a minimum

$w = 24, \quad h = 12 + \frac{384}{24-8} = 36$

3. A cone-shaped drinking cup is made from a circular piece of waxed paper of radius R by cutting out a sector, as shown, and joining the edges CA and CB. Find the maximum capacity of the cup.



Volume of a cone = $\frac{1}{3}$(area of base) height

Let $L \in (0, 2\pi R)$ be the "outer circumference" of the waxed paper

← circumference of base = L

radius of base = $\frac{L}{2\pi}$

$$h = \sqrt{R^2 - \frac{L^2}{4\pi^2}}$$

$$\text{Volume of cup} = \frac{1}{3}\pi\left(\frac{L}{2\pi}\right)^2 \sqrt{R^2 - \frac{L^2}{4\pi^2}}$$

$$V(L) = \frac{1}{12\pi} L^2 \sqrt{R^2 - \frac{L^2}{4\pi^2}}$$

$$V'(L) = \frac{1}{12\pi}\left(2L\sqrt{R^2 - \frac{L^2}{4\pi^2}} + L^2 \frac{1}{2\sqrt{R^2 - \frac{L^2}{4\pi^2}}}\left(\frac{-2L}{4\pi^2}\right)\right)$$

$$0 = \frac{\cancel{1}}{12\pi}\cancel{L}\left(\frac{2\left(R^2 - \frac{L^2}{4\pi^2}\right) - \frac{L^2}{4\pi^2}}{\cancel{\sqrt{R^2 - \frac{L^2}{4\pi^2}}}}\right)$$

$$0 = 2R^2 - \frac{3L^2}{4\pi^2}$$

$$2R^2 = \frac{3L^2}{4\pi^2}$$

$$L = 2\pi\sqrt{\frac{2}{3}}\,R$$

So $V(L) = \frac{1}{3}\pi\left(\frac{2}{3}R^2\right)\sqrt{R^2 - \frac{2}{3}R^2}$

$$= \frac{1}{3}\pi\left(\sqrt{\frac{1}{3}}\right)\frac{2}{3}R^3$$

$$V(L) = \frac{2\pi}{9\sqrt{3}}R^3$$

Note: To determine angle cut out

$$L = (2\pi - \theta)R$$

$$2\pi\sqrt{\frac{2}{3}}\,R = (2\pi - \theta)R$$

$$2\pi\sqrt{\frac{2}{3}} = 2\pi - \theta$$

$$\theta = 2\pi - 2\pi\sqrt{\frac{2}{3}} = 2\pi\left(1 - \sqrt{\frac{2}{3}}\right) \approx 67^\circ$$