Math F251: Section 4.5 Worksheet *Solutions*

Recall the guidelines:

A. domain
B. intercepts
C. symmetry
D. asymptotes
E. increase/decrease (and critical numbers)
F. local maxima/minima
G. concavity (and inflection points)
H. sketch the graph

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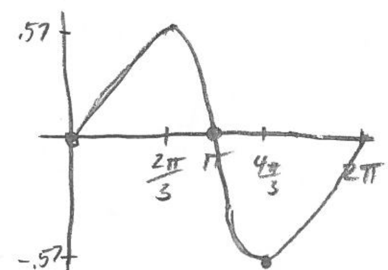
1. Sketch the graph by applying the guidelines:

$$y = \frac{\sin x}{2+\cos x}, \qquad 0 \le x \le 2\pi$$



A.

B. $x=0 \Rightarrow y=0$

$y=0 \Rightarrow x = 0, \pi, 2\pi$

C. none (period is 2π)

D. none (continuous since $2+\cos x \neq 0$)

$f\left(\frac{2\pi}{3}\right) = \frac{\sin\left(\frac{2\pi}{3}\right)}{2+\cos\left(\frac{2\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{2+\left(-\frac{1}{2}\right)}$

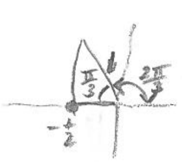
$= \frac{\sqrt{3}}{2}\cdot\frac{2}{3} = \frac{1}{\sqrt{3}} \approx .57$

$f\left(\frac{4\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

E, F. $y' = \dfrac{(\cos x)(2+\cos x) - (\sin x)(-\sin x)}{(2+\cos x)^2}$

$= \dfrac{2\cos x + \cos^2 x + \sin^2 x}{(2+\cos x)^2} = \dfrac{2\cos x + 1}{(2+\cos x)^2}$

$y' = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$



y': $+$ 0 $-$ 0 $+$ on $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$

G. $y'' = \dfrac{(-2\sin x)(2+\cos x)^2 - (2\cos x+1)(2)(2+\cos x)(-\sin x)}{(2+\cos x)^4}$

$= \dfrac{(2+\cos x)(2\sin x)[-2-\cos x + 2\cos x + 1]}{(2+\cos x)^4} = \dfrac{(2+\cos x)(2\sin x)(\cos x - 1)}{(2+\cos x)^4}$

>0 ≤ 0

concavity determined by sign of $(2\sin x)(\cos x - 1)$

y'': 0 $-$ 0 $+$ on $0, \pi, 2\pi$

2. Sketch the graph by applying the guidelines:

$$y = \frac{1}{x^2 - 4} = (x^2-4)^{-1}$$

A. domain $x \neq \pm 2$

B. $y = 0$ never

$x = 0 \quad y = -\frac{1}{4}$

C. even $f(x) = f(-x)$

D. $\lim_{x\to\infty} \frac{1}{x^2-4} = 0$

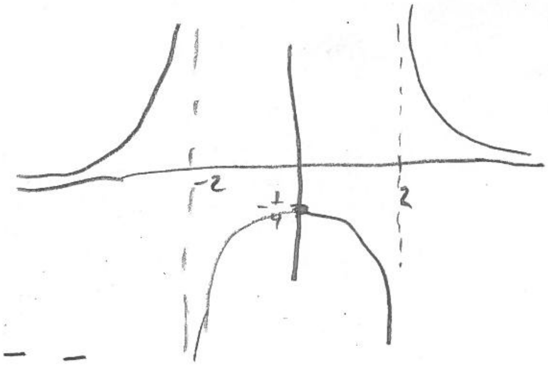
$\lim_{x\to 2^+} \frac{1}{x^2-4} = \infty$

$\lim_{x\to 2^-} \frac{1}{x^2-4} = -\infty$

E,F. $y' = \frac{-2x}{(x^2-4)^2}$ $\quad y'$: $+$ (at -2) $+$ 0 (at 0) $-$ (at 2) $-$

G. $y'' = \frac{-2(x^2-4)^2 + 2x(2)(x^2-4)(2x)}{(x^2-4)^4} = \frac{(x^2-4)(-2x^2+4+8x^2)}{(x^2-4)^4}$

$= \frac{(x^2-4)(6x^2+4)}{(x^2-4)^2}$ $\quad y''$: $+$ (at -2) $-$ (at 2) $+$



3. Sketch the graph by applying the guidelines:

$$y = \frac{x}{\sqrt{x^2+1}}$$

A. domain $(-\infty, \infty)$

B. $y = 0 \Rightarrow x = 0$

$x = 0 \Rightarrow y = 0$

C. odd: $-f(x) = f(-x)$

D. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 1$

E,F. $y' = \frac{\sqrt{x^2+1} - x\frac{1}{2}(x^2+1)^{-1/2}\,2x}{x^2+1}$

$= \frac{(x^2+1) - x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{1}{(x^2+1)^{3/2}} > 0$ always

G. $y'' = -\frac{3}{2}(x^2+1)^{-5/2}(2x)$

$= \frac{-3x}{(x^2+1)^{5/2}}$ $\quad y''$: $+$ 0 (at 0) $-$