 Solutions

1. Differentiate the functions.

$y = (1-x^2)\ln(1+x^2)$

$y' = -2x\ln(1+x^2) + \frac{(1-x^2)}{1+x^2}2x$

$y = \tan[\ln(ax+b)]$

$\frac{dy}{dx} = \sec^2(\ln(ax+b))\frac{1}{ax+b}a$

$= \frac{a\sec^2(\ln(ax+b))}{ax+b}$

$g(t) = \frac{\ln t}{\arcsin(t^2)+1}$

$g'(t) = \frac{\frac{1}{t}(\arcsin(t^2)+1) - (\ln t)\frac{1}{\sqrt{1-(t^2)^2}}}{(\arcsin(t^2)+1)^2}$

2. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2} = GmMr^{-2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) Find dF/dr and explain its meaning. What does the minus indicate?

$\frac{dF}{dr} = \frac{-2GmM}{r^3}$ tells us the rate at which the force changes as the objects move apart. The minus sign indicates the force gets smaller as the objects move apart.

(b) Assume we measure mass in kilograms, distance in meters, and force in Newtons. What are the units of dF/dr?

$\frac{N}{m}$ $\left(\frac{\text{Newtons}}{\text{meter}}\right)$

(c) Find dF/dm and explain its meaning and units.

$\frac{dF}{dm} = \frac{GM}{r^2}$ tells us the rate the force changes by if the mass of one object is increased. Its units are $\frac{N}{kg}$ s.

3. A tank holds 5000 gallons of water which drains from the bottom of the tank in 40 minutes. The volume of water remaining in the tank after t minutes is

$$V = 5000\left(1 - \frac{1}{40}t\right)^2$$

for $0 \le t \le 40$. Find the rate at which water is draining from the tank after (a) 5 min, (b) 20 min, and (c) 40 min. Which is fastest/slowest?

$\frac{dV}{dt} = 1000\left(1-\frac{1}{40}t\right)\left(-\frac{1}{40}\right) = -25\left(1-\frac{1}{40}t\right)$ $\frac{\text{gallons}}{\text{minute}}$

$\frac{dV}{dt}(5) = -25\left(1-\frac{5}{40}\right) = -25\left(\frac{7}{8}\right)$ ← fastest

$\frac{dV}{dt}(20) = -25\left(1-\frac{20}{40}\right) = -25\left(\frac{1}{2}\right)$

$\frac{dV}{dt}(40) = -25\left(1-\frac{40}{40}\right) = 0$ ← slowest