Math F251: Section 3.4 Worksheet Solutions

1. Find an equation of the tangent line to the curve at the given point.

$y = \sqrt{1+x^3}, \quad (2,3)$

$y' = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)$

$y'\big|_{x=2} = \frac{1}{2}(9)^{-1/2}(3\cdot 4) = \frac{1}{2}\cdot\frac{1}{3}\cdot 3\cdot 4 = 2$

$y - 3 = 2(x-2)$

$y = 3 + 2(x-2)$

$y = 2x - 1$

2. If $F(x) = f(g(x))$, and if $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

$F'(5) = f'(g(5))\, g'(5)$

$= f'(-2)\cdot 6$

$= 4\cdot 6 = 24$

3. Find the 49th derivative of $f(x) = x\,e^{-x}$.

$f'(x) = e^{-x} - xe^{-x}$

$f''(x) = -e^{-x} - (e^{-x} - xe^{-x})$

$= -2e^{-x} + xe^{-x}$

$f'''(x) = 2e^{-x} + (e^{-x} - xe^{-x})$

$= 3e^{-x} - xe^{-x}$

$f^{(4)}(x) = -3e^{-x} - (e^{-x} - xe^{-x})$

$= -4e^{-x} + xe^{-x}$

$f^{(49)}(x) = 49e^{-x} - xe^{-x}$

signs determined by oddness of 49

4. Find the derivative of the function. You do not need to simplify your answer.

(a) $y = \left(x + \frac{1}{x}\right)^7$

$y' = 7\left(x + \frac{1}{x}\right)^6\left(1 - \frac{1}{x^2}\right)$

(b) $f(\theta) = \cos(\theta^2)$

$f'(\theta) = -\sin(\theta^2)\,2\theta = -2\theta\sin(\theta^2)$

(c) $g(t) = 2^{t^3} = \left(e^{\ln 2}\right)^{t^3} = e^{t^3 \ln 2}$

$\frac{dg}{dt} = e^{t^3\ln 2}(3\ln 2)t^2 = (3\ln 2)t^2 2^{t^3}$

(d) $y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \left(x + \left(x + x^{1/2}\right)^{1/2}\right)^{1/2}$

$y' = \frac{1}{2}\left(x + \left(x + x^{1/2}\right)^{1/2}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x + x^{1/2}\right)^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)\right)$