1. Differentiate the following.

a. $F(r) = \frac{5}{r^3}$ $= 5r^{-3}$ $\quad F'(r) = -15r^{-4} = \frac{-15}{r^4}$

b. $y = 3e^x + \frac{4}{\sqrt[3]{x}}$ $= 3e^x + 4x^{-\frac{1}{3}}$ $\quad \frac{dy}{dx} = 3e^x - \frac{4}{3}x^{-\frac{4}{3}}$

c. $G(q) = (1 + q^{-1})^2$ $= 1 - 2q^{-1} + q^{-2}$ $\quad \frac{dG}{dq} = 2q^{-2} - 2q^{-3}$

d. $g(x) = (x + 5\sqrt{x})e^x$ $= (x + 5x^{\frac{1}{2}})e^x$

$$g'(x) = (1 + \tfrac{5}{2}x^{-\frac{1}{2}})e^x + (x + 5x^{\frac{1}{2}})e^x$$
$$= \left(1 + x + 5\sqrt{x} + \frac{5}{2\sqrt{x}}\right)e^x$$

e. $y = \frac{\sqrt{x}}{2+x}$

$$y' = \frac{\frac{1}{2}x^{-\frac{1}{2}}(2+x) - \sqrt{x}(1)}{(2+x)^2} = \frac{\frac{1}{2\sqrt{x}}(2+x) - \sqrt{x}}{(2+x)^2}$$

f. $f(x) = \frac{ax+b}{cx+d}$

$$\frac{df}{dx} = \frac{a(cx+d) - (ax+b)c}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$$

2. Find the first and second derivatives of the function.

$G(r) = \sqrt{r} - \sqrt[3]{r}$ $= r^{1/2} - r^{1/3}$

$$G'(r) = \frac{1}{2}r^{-1/2} - \frac{1}{3}r^{-2/3}$$

$$G''(r) = -\frac{1}{4}r^{-3/2} + \frac{2}{9}r^{-5/3}$$

3. Find an equation of the tangent line to the graph of $y = x^2 + 2e^x$ at $(0, 2)$.

$y' = 2x + 2e^x$

$y'\big|_{x=0} = 2$

$y - 2 = 2(x - 0)$

$y = 2 + 2x$

4. The equation of motion of a particle is $s = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration as functions of t.

$v(t) = s'(t) = 4t^3 - 6t^2 + 2t - 1$

$a(t) = v'(t) = s''(t) = 12t^2 - 12t + 2$

(b) Find the acceleration at time 1 s.

$a(1) = 12 - 12 + 2 = 2 \frac{m}{sec^2}$

5. A quantity p of fabric, measured in yards, is sold at a price $f(p)$ (dollars) which depends on the quantity. The total revenue from a sale of p yards of fabric is $R(p) = pf(p)$.

(a) What does it mean to say that $f(20) = 100$ and that $f'(20) = -0.5$?

When 20 yds are sold the price is \$100 per yard.
When 20 yds are sold the price declines at a rate of \$.50 for additional yards

(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

$R'(p) = \frac{d}{dp}(pf(p)) = 1 \cdot f(p) + p \cdot f'(p)$

So $R'(20) = f(20) + 20f'(20) = 100 + 20 \cdot (-.5) = 90$ \$

When 20 yds are sold, the revenue increases by \$90 for additional yds.