

1. Find $f'(a)$ using the definition of the derivative:

$$f(t) = 2t^2 + t$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{2(a+h)^2 + (a+h) - (2a^2 + a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{a} + h - \cancel{2a^2} - \cancel{a}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h}(4a + 2h + 1)}{\cancel{h}} = 4a + 1 \end{aligned}$$

2. Find $f'(3)$ using the definition of the derivative:

$$f(x) = x^{-2}$$

$$\begin{aligned} f'(3) &= \lim_{h \rightarrow 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h} = \lim_{h \rightarrow 0} \frac{9 - (3+h)^2}{9(3+h)^2 h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{9} - (\cancel{9} + 6h + h^2)}{9(3+h)^2 h} = \lim_{h \rightarrow 0} \frac{-\cancel{h}(6+h)}{9(3+h)^2 \cancel{h}} = \frac{-6}{81} = \frac{-2}{27} \end{aligned}$$

3. Find $f'(a)$ using the definition of the derivative:

$$f(x) = \sqrt{1+5x}$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \left(\frac{\sqrt{1+5(a+h)} - \sqrt{1+5a}}{h} \right) \left(\frac{\sqrt{1+5(a+h)} + \sqrt{1+5a}}{\sqrt{1+5(a+h)} + \sqrt{1+5a}} \right) \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h}(1+5(a+h)) - \cancel{h}(1+5a)}{h(\sqrt{1+5(a+h)} + \sqrt{1+5a})} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{h}5}{h(\sqrt{1+5(a+h)} + \sqrt{1+5a})} = \frac{5}{2\sqrt{1+5a}} \end{aligned}$$

4. Find an equation of the tangent line to the curve at the given point:

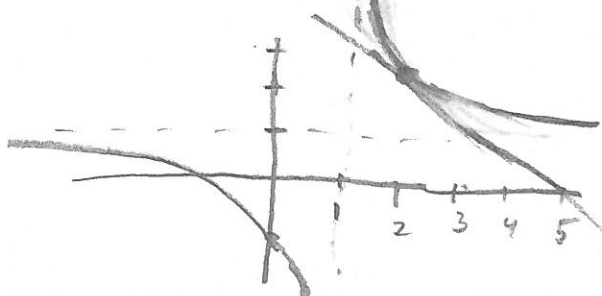
$$f(x) = \frac{x+1}{x-1}, \quad (2, 3)$$

Also sketch both the curve $y = f(x)$ and the tangent line.

$$f'(2) = \lim_{h \rightarrow 0} \frac{\frac{(2+h)+1}{(2+h)-1} - 3}{h} = \lim_{h \rightarrow 0} \frac{3+h - 3(1+h)}{(1+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{(1+h)h} = -2$$

tangent line: $y - 3 = -2(x - 1)$ or $y = 3 - 2(x - 1)$
or $y = 5 - 2x$



5. A particle moves a distance $s = f(t)$ along a straight line, where s is measured in meters and t is in seconds:

$$f(t) = 40t - 5t^2$$

Find the velocity and speed when $t = 4$.

$$\text{velocity} = f'(4) = \lim_{h \rightarrow 0} \frac{40(4+h) - 5(4+h)^2 - (40 \cdot 4 - 5(4)^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{40 \cdot 4} + 40h - 5(4^2 + 2 \cdot 4 \cdot h + h^2) - \cancel{40 \cdot 4} + 5(4)^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(40 - 40 - 5h)}{\cancel{h}} = \lim_{h \rightarrow 0} -5h = 0 \frac{\text{m}}{\text{sec}}$$

$$\text{speed} = |f'(4)| = \cancel{20} \frac{\text{m}}{\text{sec}}$$