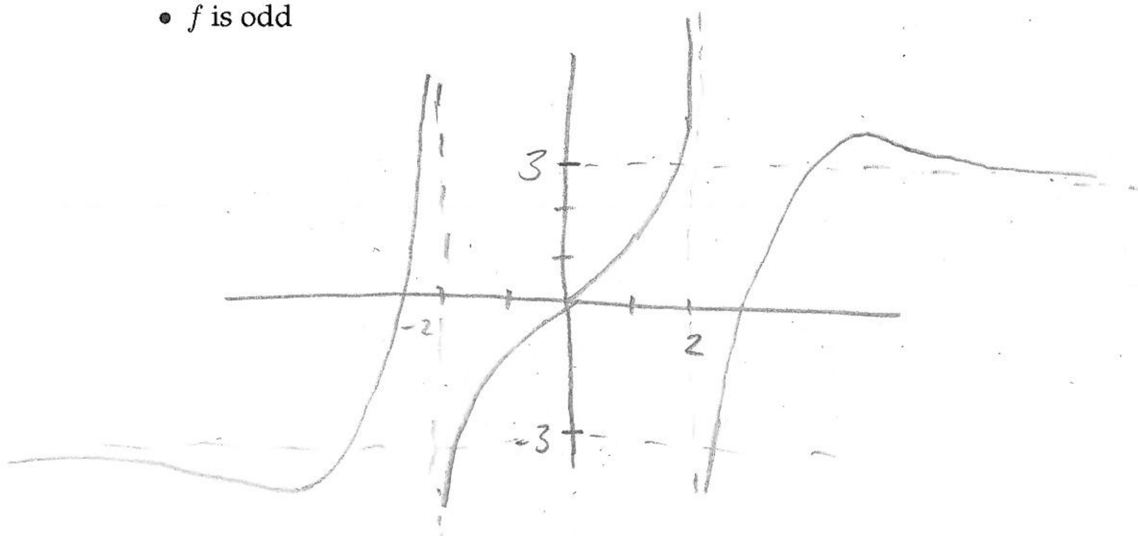
1. Sketch the graph of a function that satisfies all of the given conditions:

- $\lim_{x\to\infty} f(x) = 3$
- $\lim_{x\to 2^-} f(x) = \infty$
- $\lim_{x\to 2^+} f(x) = -\infty$
- f is odd

One possible answer:



2. Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2},$$

and clearly state limits which justify these asymptotes.

(Along the way you make very rough sketch of the graph. You may be able to confirm your work by graphing calculator.)

Horizontal: $y = 2$ $\quad \lim_{x\to\infty} \frac{2x^2+x-1}{x^2+x-2} = 2$

$\lim_{x\to-\infty} \frac{2x^2+x-1}{x^2+x-2} = 2$

Vertical: $x = -2$, $x = 1$ $\quad \frac{2x^2+x-1}{x^2+x-2} = \frac{2x^2+x-1}{(x+2)(x-1)} = f(x)$

$\lim_{x\to -2^-} f(x) = \infty \qquad \lim_{x\to 1^-} f(x) = -\infty$

$\lim_{x\to -2^+} f(x) = -\infty \qquad \lim_{x\to 1^+} f(x) = \infty$

3. Show that f is continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

Since $\sin x$, $\cos x$ are continuous everywhere, we just need to see that $\lim_{x \to \pi/4} f(x) = f(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

Since $\frac{1}{\sqrt{2}} = \sin(\pi/4) = \cos(\frac{\pi}{4})$

we know $\lim_{x \to \frac{\pi}{4}^-} \sin x = \frac{1}{\sqrt{2}}$ and $\lim_{x \to \frac{\pi}{4}^+} \cos x = \frac{1}{\sqrt{2}}$

So $\lim_{x \to \pi/4} f(x) = \frac{1}{\sqrt{2}}$.

4. Prove that the equation has at least one real root:

$$\ln x = 3 - 2x$$

(You may use a calculator to find an accurate approximation, but this is not required.)

Consider $f(x) = \ln x - 3 + 2x$, which is continuous for $x > 0$

$f(1) = \ln 1 - 3 + 2 = -1$

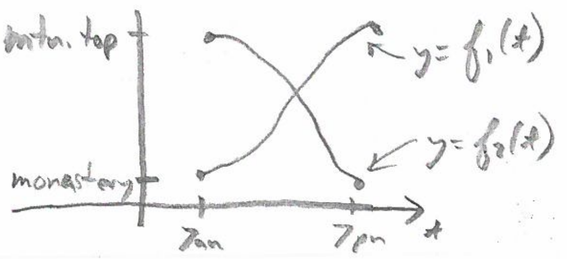
$f(e) = \ln e - 3 + 2e = 1 - 3 + 2(2.71\ldots) > 0$

So $f(c) = 0$ for some $c \in (1, e)$

So $\ln c = 3 - 2c$ for some $c \in (1, e)$

5. *A challenge problem, but actually easy. It follows from the Intermediate Value Theorem. Start by sketching elevation versus time for each day, one on top of the other.*

A Tibetan monk leaves the monastery at 7:00 AM and takes his usual parth to the top of the mountain, arriving at 7:00 PM and sleeping on top. The next morning he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 AM. Show that there is a point on the path that the monk will cross at exactly the same time of day on both days.



Let $g(t) = f_1(t) - f_2(t)$ which is continuous

$g(7am) < 0$, $g(7pm) > 0$

So $g(c) = 0$ for some $c \Rightarrow f_1(c) = f_2(c)$.