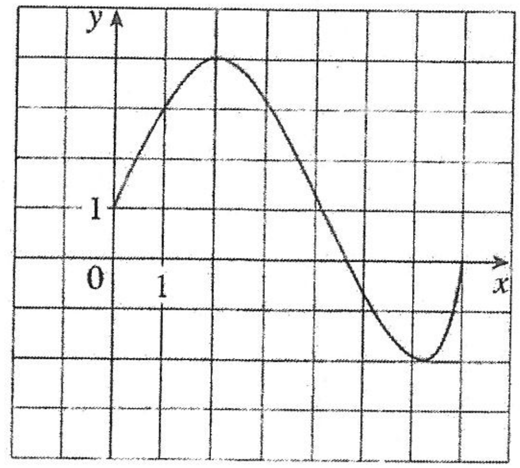
 Solutions

1. The graph of a function f is shown below. Find the following:



a) $f(1)$ and $f(5)$

$f(1)=3, \; f(5)=-.6$

b) the domain of f

$[0,7]$

c) the range of f

$[-2,4]$

d) For which value of x is $f(x) = 4$?

$x=2$

e) Where is f increasing?

$[0,2] \cup [6.25, 7]$

2. Let $f(x) = 3x^2 - x + 2$. Find and simplify the following expressions.

(a) $f(2) = 3 \cdot 2^2 - 2 + 2 = 12$

(b) $f(a^2) = 3(a^2)^2 - a^2 + 2 = 3a^4 - a^2 + 2$

(c) $[f(a)]^2 = (3a^2 - a + 2)^2 = 9a^4 - 6a^3 + (12a^2 + (-a)^2) - 4a + 4$

$= 9a^4 - 6a^3 + 13a^2 - 4a + 4$

(d) $$\frac{f(2+h) - f(2)}{h} = \frac{3(2+h)^2 - (2+h) + 2 - (3(2)^2 - 2 + 2)}{h}$$

$$= \frac{3(4+4h+h^2) - \cancel{2} - h + \cancel{2} - 12 + \cancel{2} - \cancel{2}}{h} = \frac{11h + 3h^2}{h} = 11 + 3h$$

(e) $$\frac{f(a+h) - f(a)}{h} = \frac{3(\cancel{a^2} + 2ah + h^2) - (\cancel{a} + h) + \cancel{2} - (\cancel{3a^2} - \cancel{a} + \cancel{2})}{h}$$

$$= \frac{6ah + 3h^2 - h}{h} = 6a - 1 + 3h$$

3. Find the domain of each of the following functions. Use interval notation.

1. $f(x) = \dfrac{1}{x^4 - 16}$

Need $x^4 - 16 \neq 0$

$x^4 \neq 16$

$x \neq \pm 2$

$(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

2. $f(x) = \sqrt{x} + \sqrt{11-x}$

Need $x \geq 0$ $\quad 11 - x \geq 0$

$11 \geq x$

$[0, 11]$

3. $g(x) = \ln(x-4)$

Need $x - 4 > 0$

$x > 4$

$(4, \infty)$

4. $h(x) = \frac{1}{\sqrt{x^2-5x-6}}$

Need $x^2 - 5x - 6 > 0$

$(x-6)(x+1) > 0$

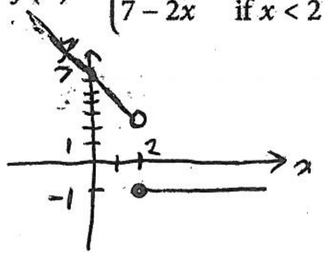
So either $x-6>0,\ x+1>0$ i.e. $x>6$

or $x-6<0,\ x+1<0$ i.e. $x<-1$

$(-\infty,-1) \cup (6,\infty)$

4. Graph each of the following piecewise defined functions.

a) $f(x) = \begin{cases} -1 & \text{if } x \geq 2 \\ 7 - 2x & \text{if } x < 2 \end{cases}$



b) $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$