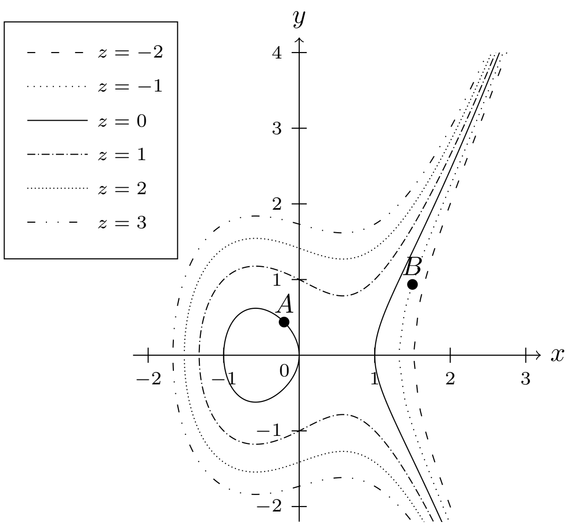
MATH202X-F01/FUX1 Midterm Exam 2 Name:______________________

Spring 2015

Instructions. (100 points) You have 60 minutes. No calculators allowed. *Show all your work* in order to receive full credit.

(6^{pts}) **1.** Explain why $\lim\limits_{(x,y)\to(1,0)} \dfrac{xy^2}{(x-1)^2+y^2}$ does not exist.

(6^{pts}) **2.** The plot below shows several level curves of a function $z = f(x, y)$. At the points A and B sketch vectors representing the correct directions for ∇f. Would ∇f be longer at A or at B?



(6^{pts}) **3.** The pressure P (in kilopascals) of one mole of an ideal gas is determined by its temperature T (in kelvins) and volume V (in liters) according to

$$P = 8.3\frac{T}{V}.$$

If $T = 300$ kelvins, $dT/dt = 0.2$ kelvins/sec, $V = 10$ liters, $dV/dt = 0.1$ liters/sec, at what rate will the pressure be changing?

(12$^{\text{pts}}$) **4.** Compute the iterated integral by switching the order of integration.

$$I = \int_0^1 \int_{-1}^{-\sqrt{y}} e^{x^3}\, dx\, dy.$$

(8$^{\text{pts}}$) **5.** Using the method of Lagrange multipliers, find the points on the circle $x^2 + y^2 = 1$ where the maxima and minima of the function

$$f(x, y) = x^2 + y$$

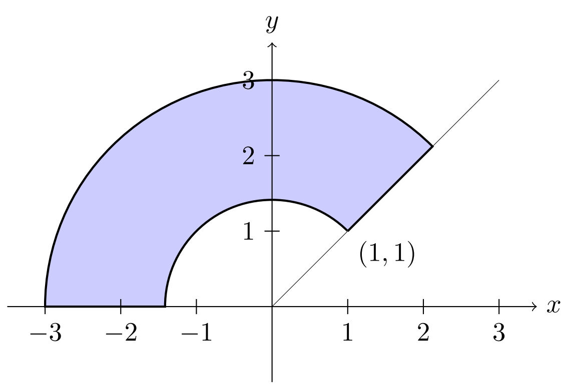
occur. For each of the points, indicate whether a maximum or a minimum occurs.

(8pts) **6.** Find an equation of the tangent plane to the surface

$$y\cos(z+x)+z^2=5$$

at the point $(x_0, y_0, z_0) = (2, 0, -2)$.

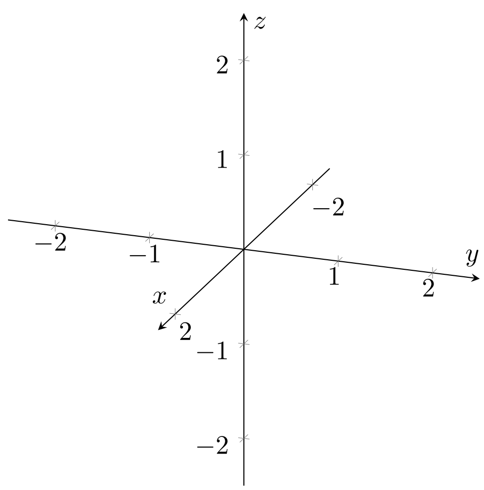
(10pts) **7.** Compute $\iint_R f(x,y)\,dA$ for $f(x,y)=\sqrt{9-x^2-y^2}$ and R the bounded region shaded below.



(12[pts]) **8.** The mass of a solid Q is given by:

$$m = \int_0^2 \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\pi} \rho^4 \cos^2\phi \sin\phi \, d\phi \, d\theta \, d\rho.$$

(a) (4 pts) Describe and sketch the solid Q.



(b) (2 pts) Deduce from the equation above the density function:

$f(x, y, z) =$

(c) (6 pts) Evaluate the mass m.

(20pts) **9.** Consider the function

$$f(x, y) = x^2 + xy + y^3 + 2.$$

(a) (4 pts) At the point $(2, -1)$, in which direction should you move to produce the greatest rate of *decrease* in f?

(b) (6 pts) At the point $(2, -1)$, what is the directional derivative of f in the direction towards the origin?

(c) (4 pts) Show that $(0, 0)$ and $(-1/12, 1/6)$ are critical points of f. (They are the only critical points, but you need not show that.)

(d) (6 pts) Determine whether each of the critical points is a maximum, a minimum, or a saddle point.

(12pts) **10.** Let $(\overline{x}, \overline{y})$ be the center of mass of a triangular planar lamina of density $\rho(x, y) = y$ determined by the vertices $(-1, 0)$, $(1, 0)$, and $(0, 2)$.

(a) (9 pts) Write an integral formula for $\overline{x}$. Fully set up the integral(s) with the integrand and limits of integration, but DO NOT EVALUATE.

(b) (3 pts) Can you find the value of $\overline{x}$ without computation? Explain your answer.