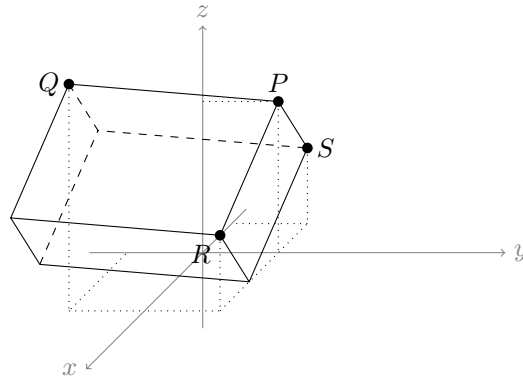


Instructions. You have 60 minutes. No calculators allowed. *Show all your work* in order to receive full credit.

1. Consider the points $P(0, 1, 2)$, $Q(2, -1, 3)$, $R(2, 1, 1)$, and $S(-1, 1, 1)$ in space.



- (a) Give the equation of the plane containing the points P , Q , and R .

- (b) Find the volume of the parallelepiped with vertices P , Q , R and S as drawn above.

2. Consider the following planes in space:

$$\text{Plane 1} \quad x + y - z - 2 = 0$$

$$\text{Plane 2} \quad 2x - y + z - 1 = 0$$

(a) Find the point with y -coordinate equal to zero which lies on both planes.

(b) Give parametric equations for the line of intersection of the two planes.

3. Let $\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$ be describing the motion of a particle along a plane curve over time.

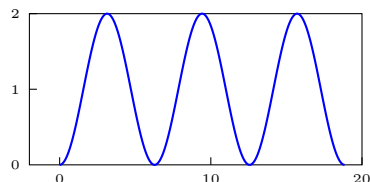
(a) Find all the open intervals on which the curve is smooth.

(b) What is the speed of the particle at $t = \frac{\pi}{2}$?

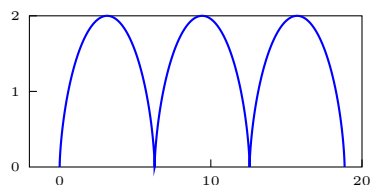
(c) Use smoothness to match the following plane curves to the correct graphs below.

$$\vec{r}_1(t) = \langle t - \sin t, 1 - \cos t \rangle \quad , \quad \vec{r}_2(t) = \langle t, 1 - \cos t \rangle \quad , \quad \vec{r}_3(t) = \langle 2t - \sin(2t), 1 - \cos t \rangle$$

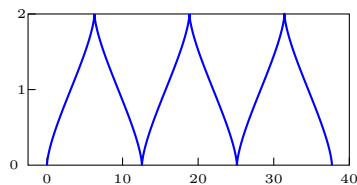
Briefly justify your answers. Each curve corresponds to $0 \leq t \leq 6\pi$ (and $x(t)$ is represented along the horizontal axis).



(a) _____



(b) _____

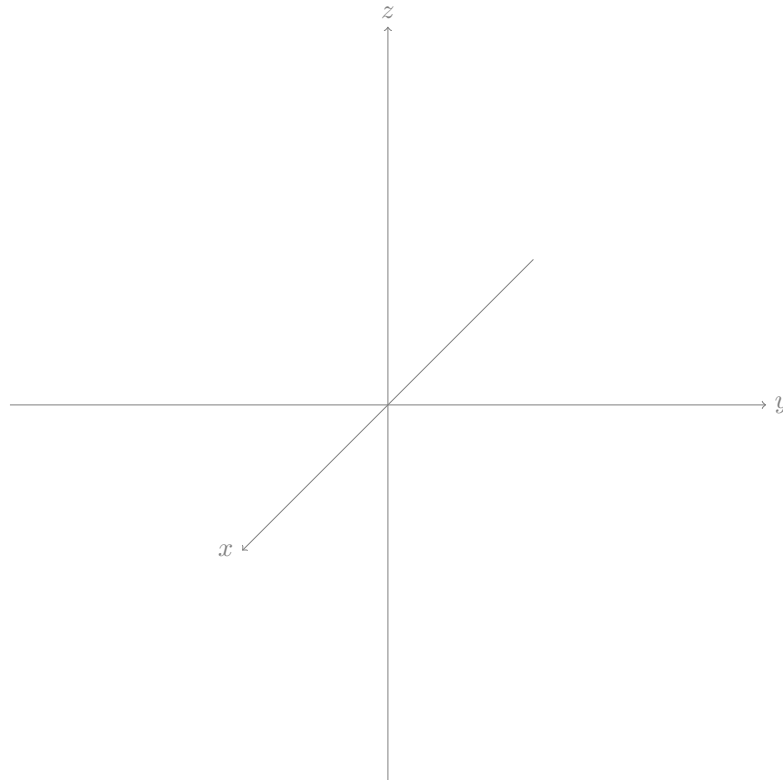


(c) _____

4. Time for some coordinate conversions!

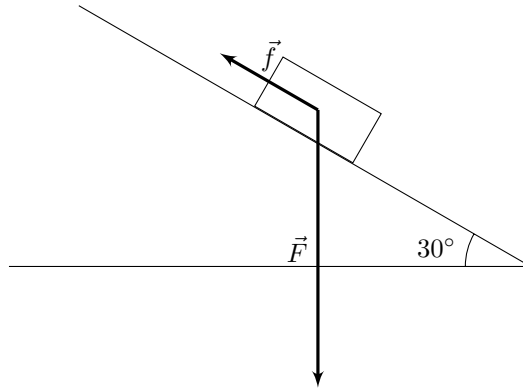
(a) Convert the point P with cylindrical coordinates $P\left(3, \frac{\pi}{6}, 1\right)$ to rectangular coordinates.

(b) Describe and sketch the surface in space whose equation in spherical coordinates is given by $\rho = 2$.



5. Little Jimmy is getting ready to go down the sledding hill, but he needs to adjust his hat and scarf first. So he asks his little sister Tina to apply tension to the string by holding it in the same inclination as the hill.

- (a) The force she exerts on it is represented by \vec{f} , and the force of gravity from Jimmy and his sled is represented by \vec{F} in the sketch below. We assume no friction is present. Sketch $\text{proj}_{\vec{f}} \vec{F}$.



- (b) Assume the mass of little Jimmy and his sled together is $m = 50\text{kg}$ and that Tina exerts a force of magnitude 200N . The hill is at an angle of 30° with the horizontal. Compute $\text{proj}_{\vec{f}} \vec{F}$. (Use $g = 9.8\text{m/s}^2$ for the magnitude of gravity.)

- (c) Based on your drawing and/or your result above, what will happen to the sled?

6. Given the symmetry equations of two lines representing the trajectories of two particles (with t measured in seconds):

$$\text{Line 1: } x + 1 = \frac{y}{2} = \frac{z - 1}{-1}$$

$$\text{Line 2: } \frac{x - 2}{-3} = y + 1 = \frac{z + 3}{4}$$

- (a) Show that the graphs of the lines intersect at the point $(-1, 0, 1)$. Do the two particles collide at $(-1, 0, 1)$? Explain your answer.

- (b) Find the distance from $(-1, 0, 1)$ to Plane 1 from problem 2: $x + y - z - 2 = 0$.

7. In the plane a particle moves with acceleration:

$$\vec{a}(t) = \pi^2 \cos(\pi t)\vec{i} + 3t^2\vec{j}.$$

At time $t = 1$, its velocity is $\vec{v}(1) = \vec{i} - \vec{j}$. At time $t = 0$, its position is $\vec{r}(0) = \vec{i} + \vec{j}$. Find the position at time $t = 2$.

8. An object moves along a trajectory so that its position $\vec{r}(t)$ as a function of time is given by:

$$\vec{r}(t) = \left\langle t^2, \frac{2t^3}{3}, t \right\rangle.$$

Find the tangential and normal components of the acceleration for the object. (*Hint:* Use a dot product formula for the tangential component, and deduce the normal one from $a_{\vec{N}} = \sqrt{\|\vec{a}\|^2 - a_{\vec{T}}^2}$.)

9. In this problem, we deal with the same object as in the previous problem, that is whose trajectory can be described by the position vector:

$$\vec{r}(t) = \left\langle t^2, \frac{2t^3}{3}, t \right\rangle.$$

You should be able to use some of your work for these new questions.

- (a) Find the arc length function of the trajectory (assume $t \geq 0$), and use it to find the distance traveled between $t = 0$ and $t = 2$ along the trajectory.

- (b) Find the curvature K of the trajectory at time t using the formula of your choice.

10. A projectile is fired from ground level at an angle θ with the horizontal. The projectile is to have a range of L units. Find the initial speed necessary. (Your final expression will contain θ and L .)

