

1. (15 pts.-5 pts. each) A plane is given by the equation $3x - 2y + 5z = 1$.

(a) Give an equation for the parallel plane through the point $(3, 1, 2)$.

Alternate solution:

$$3(x-3) - 2(y-1) + 5(z-2) = 0$$

Parallel planes have equations $3x - 2y + 5z = d$.
To pick d so $(3, 1, 2)$ is on the plane we need $3 \cdot 3 - 2 \cdot 1 + 5 \cdot 2 = d$
 $17 = d$
So $3x - 2y + 5z = 17$

(b) Give a parameterization of the line through the point $(3, 1, 2)$ that is orthogonal to the plane.

direction $\langle 3, -2, 5 \rangle$
point $(3, 1, 2)$

$$\vec{r}(t) = \langle 3+3t, 1-2t, 2+5t \rangle$$

(c) What is the angle between this plane and the xz -coordinate plane?
(Your answer may involve an inverse trigonometric function.)

normal to plane above: $\vec{n} = \langle 3, -2, 5 \rangle$
normal to xz -plane: $\vec{m} = \langle 0, 1, 0 \rangle$

$$\|\vec{n}\| \|\vec{m}\| \cos \theta = \vec{n} \cdot \vec{m}$$

$$\sqrt{9+4+25} \cdot 1 \cos \theta = -2$$

$$\cos \theta = \frac{-2}{\sqrt{38}} = -\sqrt{\frac{2}{19}}$$

$$\theta = \cos^{-1}\left(-\sqrt{\frac{2}{19}}\right)$$

Note: $\theta = \cos^{-1}\left(\sqrt{\frac{2}{19}}\right)$ is also correct, since the problem did not specify the acute or obtuse angle

2. (14 pts.-7 pts. each) Due to gravity, an object that weighs 5 N slides down a straight frictionless ramp from the point $(0, 0, 10)$ m to the point $(0, 30, 0)$ m.

(a) How much work was done by gravity? (Include appropriate units in your answer.)

$$\vec{F} = \langle 0, 0, -5 \rangle \text{ N}$$

$$\vec{d} = \langle 0, 30, 0 \rangle - \langle 0, 0, 10 \rangle = \langle 0, 30, -10 \rangle \text{ m}$$

$$W = \vec{F} \cdot \vec{d} = 50 \text{ Nm}$$

- (b) Give a vector describing a force along the ramp that would have prevented the object from moving.

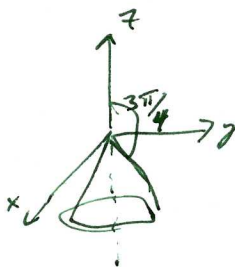
$$-\text{proj}_{\vec{d}} \vec{F} = - \frac{\vec{F} \cdot \vec{d}}{\vec{d} \cdot \vec{d}} \vec{d} = - \frac{50}{0+900+100} \langle 0, 30, -10 \rangle$$

This opposes the part of \vec{F} that points along the ramp

$$= \langle 0, -\frac{3}{2}, \frac{1}{2} \rangle \text{ N}$$

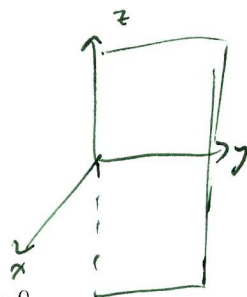
3. (20 pts.-5 pts. each) Roughly sketch the following surfaces in 3-d, given by equations in various coordinate systems. Include the x -, y -, and z -axes in each sketch.

(a) $\phi = \frac{3\pi}{4}$



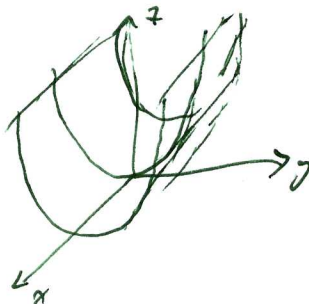
cone opening downward

(b) $\theta = \frac{\pi}{2}$



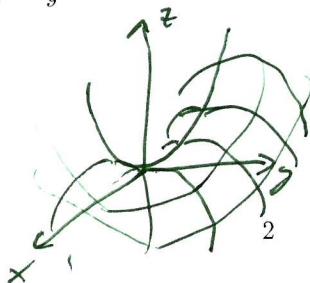
$y-z$ half-plane with $y \geq 0$

(c) $y^2 - z = 0$



parabolic cylinder

(d) $z = x^2 - y^2$



saddle (up in $\pm x$ directions down in $\pm y$ directions)

4. (14 pts.) An object moves through space with acceleration vector $\mathbf{a}(t) = \langle t, \pi \sin(\pi t), -2 \rangle$ m/sec². At time $t = 0$, its velocity is $\mathbf{v}(0) = \langle 0, 0, 4 \rangle$.

- (a) (7 pts.) Find the object's velocity $\mathbf{v}(t)$, as a function of time.

$$\vec{v}(t) = \left\langle \frac{t^2}{2} + c_1, -\cos(\pi t) + c_2, -2t + c_3 \right\rangle$$

$$\vec{v}(0) = \langle c_1, -1 + c_2, c_3 \rangle = \langle 0, 0, 4 \rangle$$

So

$$\vec{v}(t) = \left\langle \frac{t^2}{2}, 1 - \cos(\pi t), 4 - 2t \right\rangle$$

- (b) (3 pts.) What is the object's speed at time $t = 1$?

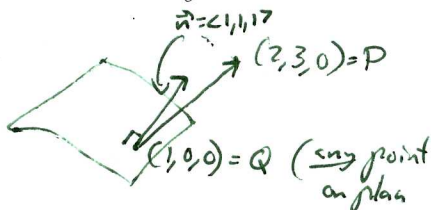
$$\vec{v}(1) = \left\langle \frac{1}{2}, 1 - (-1), 4 - 2 \right\rangle = \left\langle \frac{1}{2}, 2, 2 \right\rangle$$

$$\|\vec{v}(1)\| = \sqrt{\frac{1}{4} + 4 + 4} = \sqrt{\frac{33}{4}} = \left(\frac{\sqrt{33}}{2} \frac{\text{m}}{\text{sec}} \right)$$

- (c) (4 pts.) Give an integral for the total distance the object travels between time $t = 0$ and $t = 4$. (Do not evaluate the integral, but leave it in a form where only single-variable calculus is needed to understand it.)

$$\int_0^4 \|\vec{v}(t)\| dt = \int_0^4 \sqrt{\left(\frac{t^2}{2}\right)^2 + (1 - \cos(\pi t))^2 + (4 - 2t)^2} dt$$

5. (11 pts.) Find the distance between the point $(2, 3, 0)$ and the plane $x + y + z = 1$.



$$\text{proj}_{\vec{n}} \vec{QP} = \frac{\vec{n} \cdot \vec{QP}}{\vec{n} \cdot \vec{n}} \vec{n}$$

$$= \frac{\langle 1, 1, 1 \rangle \cdot \langle 1, 3, 0 \rangle}{\langle 1, 1, 1 \rangle \cdot \langle 1, 1, 1 \rangle} \langle 1, 1, 1 \rangle$$

$$= \frac{4}{3} \langle 1, 1, 1 \rangle$$

$$\text{so } \|\text{proj}_{\vec{n}} \vec{QP}\| = \left(\frac{4}{3} \sqrt{3} \right)$$

