Quiz 4 Name: Solutions

Math 202 F01 02/11/15

1. The shape of a wire in 3-dimensional space is described by

$$\mathbf{r}(t) = \langle t + \cos(\pi t), 3 - t^2, \ln(1+t)\rangle \quad 0 \le t \le 5.$$

Find the unit tangent vector to the wire at position $(0, 2, \ln 2)$.

Note $\vec{r}(1) = <0, 2, \ln 2>$

$\vec{r}'(t) = < 1 - \pi \sin(\pi t), -2t, \frac{1}{1+t} >$

$\vec{r}'(1) = < 1 - \pi \sin \pi, -2, \frac{1}{2} > = < 1, -2, \frac{1}{2} >$

$||r'(1)|| = \sqrt{1+4+\frac{1}{4}} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$

$\vec{T}(1) = \frac{\vec{r}'(1)}{||\vec{r}'(1)||} = \frac{2}{\sqrt{21}} < 1, -2, \frac{1}{2} > = < \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}, \frac{1}{\sqrt{21}} >$

2. An object (near the surface of some planet other than earth) moves with constant acceleration due to gravity of $\langle 0, 0, -2\rangle\, m/sec^2$. At time $t = 2\,sec$ it has position $(-3, -5, 10)\, m$ and velocity $\langle 1, 2, 0\rangle\, m/sec$. Give a parameterization of the trajectory it follows, as a function of time.

$r''(t) = <0, 0, -2>$

so $r'(t) = < c, d, -2t + e >$

but $r'(2) = <1, 2, 0> = <c, d, -4+e>$ shows $c=1, d=2, e=4$

$r'(t) = < 1, 2, 4 - 2t >$

So $r(t) = < t + c, 2t + d, 4t - t^2 + e >$

but $r(2) = <-3, -5, 10> = < 2 + c, 4 + d, 8 - 4 + e >$

shows $c = -5, d = -9, e = 6$

$r(t) = < t - 5, 2t - 9, 4t - t^2 + 6 >$