Quiz 3
Math 202 F01

Name: Solutions
02/03/2015

1. Two planes are given by

$$-x + 3y + z = 5,$$
$$2x - y + 2z = 3$$

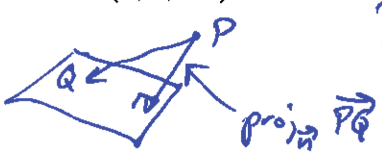
What is the angle between them? (Your answer may involve an inverse trigonometric function.)

The normal vectors to the planes are $\vec{n} = \langle -1, 3, 1\rangle$ and $\vec{m} = \langle 2, -1, 2\rangle$

So the angle is $\theta = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{m}}{\|\vec{n}\|\,\|\vec{m}\|}\right) = \cos^{-1}\left(\frac{-3}{\sqrt{11}\sqrt{9}}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{11}}\right)$

Note: This is an obtuse angle between the planes. If you prefer the acute angle, use $\theta = \cos^{-1}\left(\frac{|\vec{n}\cdot\vec{m}|}{\|\vec{n}\|\,\|\vec{m}\|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{11}}\right)$

2. What is the distance between the plane $2x - y + 2z = 3$ and the point $P(2, 1, -1)$?



Pick any point Q on the plane, say $Q(1,-1,0)$

Then $\vec{PQ} = \langle -1, -2, 1\rangle$

The projection of $\vec{PQ}$ on the plane's normal vector $\vec{n} = \langle 2, -1, 2\rangle$ is

$\text{proj}_{\vec{n}}\vec{PQ} = \frac{\vec{PQ}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n} = \frac{2}{9}\langle 2, -1, 2\rangle$

so $\|\text{proj}_{\vec{n}}\vec{PQ}\| = \frac{2}{9}\sqrt{9} = \frac{2}{3}$