Quiz 11 Name: Solutions

Math 202 F01 4/27/15

1. A parameterized surface is given by

$$\mathbf{r}(u,v) = \langle u+v, v^2, u^2\rangle,$$

with $1 \leq u \leq 2,\ 1 \leq v \leq 4$.

Set up (but do not evaluate) an integral for computing its surface area. Your answer should be left in a form where all that remains to be done is the evaluation of a double integral.

$$\vec{r}_u = \langle 1, 0, 2u\rangle$$

$$\vec{r}_v = \langle 1, 2v, 0\rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle -4uv, 2u, 2v\rangle$$

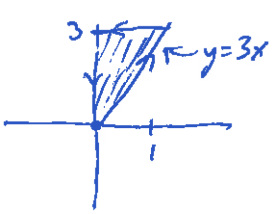
$$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{16u^2v^2 + 4u^2 + 4v^2}$$

$$= 2\sqrt{4u^2v^2 + u^2 + v^2}$$

$$\iint_S dS = \int_1^2\int_1^4 2\sqrt{4u^2v^2+u^2+v^2}\, dv\, du$$

2. Use Green's Theorem to evaluate $\int_C x^2y^2\,dx + (4xy+e^y)\,dy$ where C is the triangle with vertices $(0,0)$, $(1,3)$, and $(0,3)$, traced counterclockwise.

$$\int_C x^2y^2\,dx + (4xy+e^y)\,dy = \iint_R \frac{\partial}{\partial x}(4xy+e^y) - \frac{\partial}{\partial y}(x^2y^2)\, dA$$



$$= \iint_R (4y - 2x^2y)\, dA = \int_0^1\int_{3x}^3 (4y - 2x^2y)\, dy\, dx$$

$$= \int_0^1 (2y^2 - x^2y^2)\Big|_{y=3x}^{3}\, dx = \int_0^1 18 - 9x^2 - 18x^2 + 9x^4\ dx$$

$$= \int_0^1 18 - 27x^2 + 9x^4\ dx = 18x - \frac{27x^3}{3} + \frac{9x^5}{5}\Big|_0^1 = 18 - 9 + \frac{9}{5}$$

$$= 9 + \frac{9}{5} = \frac{54}{5}$$