Quiz 10 Name: Solutions
Math 202 F01 4/14/2015

1. Find all potential functions for the vector field

$$\mathbf{F}(x,y,z) = \langle 2xz + y^2, 2xy + z, x^2 + y + 3z^2 \rangle.$$

$\frac{\partial f}{\partial x}(x,y,z) = 2xz + y^2$, so $f(x,y,z) = x^2 z + xy^2 + C(y,z)$

Since $\frac{\partial f}{\partial y}(x,y,z) = 2xy + \frac{\partial C}{\partial y}(y,z) = 2xy + z$, we see $\frac{\partial C}{\partial y}(y,z) = z$

Thus $C(y,z) = yz + D(z)$ and $f(x,y,z) = x^2 z + xy^2 + yz + D(z)$

Since $\frac{\partial f}{\partial z}(x,y,z) = x^2 + y + \frac{dD}{dz}(z) = x^2 + y + 3z^2$, we have $\frac{dD}{dz}(z) = 3z^2$

Thus $D(z) = z^3 + E$ and $\boxed{f(x,y,z) = x^2 z + xy^2 + yz + z^3 + E}$

2. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where

$$\mathbf{F}(x,y) = \langle xy, x^2 \rangle$$

and C is parameterized by

$$\mathbf{r}(t) = \langle t^2, -t \rangle, \quad 0 \le t \le 2.$$

Note: This field is *not* conservative.

$\vec{F}(x(t), y(t)) = \langle (t^2)(-t), (t^2)^2 \rangle = \langle -t^3, t^4 \rangle$

$d\vec{r} = \vec{r}\,'(t)\,dt = \langle 2t, -1 \rangle dt$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^2 \langle -t^3, t^4 \rangle \cdot \langle 2t, -1 \rangle dt = \int_0^2 -2t^4 - t^4 \, dt$$

$$= \int_0^2 -3t^4 \, dt = \left. \frac{-3t^5}{5} \right|_0^2 = \frac{-3(2^5)}{5} = \frac{-96}{5}$$